# What: Energy Quantitative Modeling Mini CourseWhen: Monday and Tuesday, October 24 and 25, 2011Where: Room C14, WPI

# *Title:* Advanced Modeling of the Energy Spots, Forwards, Swaps and Options in the Unified Framework of the Non-Markovian Approach

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# Introduction

We present and further develop a new approach to modeling energy spot prices with spikes proposed earlier by the author. In contrast to other approaches, we model energy spot prices with spikes as a non-Markovian stochastic process that allows for modeling spikes directly as self-reversing jumps. We show how this approach can be used to model energy forwards and swaps as well as to price and hedge energy derivatives for energy spot prices with spikes. We also discuss the applications of this approach to modeling oil, natural gas, power and carbon markets.

# Day 1

# Session 1

#### **Introduction to Modeling Energy Markets**

- Overview of the existing approaches to modeling energy spots, forwards, swaps and options
- Benefits of a unified framework applicable across instruments, commodities and regions
- The non-Markovian approach as a unified framework for the consistent modeling of energy spots, forwards, swaps and options

# Session 2

#### **Modeling Energy Spots**

• The non-Markovian process for energy spots as the product of the spike and interspike processes

- Statistical and structural interpretations of the non-Markovian process for energy spots
- Taking into account daily, weekly and annual cyclical patterns, as well as trends and spikes
- Analytical probability distributions for the non-Markovian process for energy spots with the geometric mean-reverting inter-spike process and spike process with Pareto spikes
- Monte Carlo simulations of the non-Markovian process for energy spots with examples of the oil, natural gas, power and carbon markets.

# Session 3

### **Modeling Energy Forwards and Swaps**

- Consistent modeling of energy spots, forwards and swaps in the non-Markovian approach
- Taking into account daily, weekly and annual cyclical patterns, as well as trends and spikes
- Analytical modeling of energy forwards and swaps for energy spots with Pareto spikes
- Extracting the market price of risk and forward-looking market-implied riskneutral probability distributions
- Why long maturity energy forwards and swaps do not exhibit spikes while energy spots do?
- How important are spikes in modeling energy forwards and swaps?
- Modeling energy forward and swap curves with examples of oil, natural gas, power and carbon markets.

# Session 4

# Pricing Single-Commodity European Options on Energy Spots

- Pricing European options on energy spots with spikes in the non-Markovian approach
- Linear evolution equation for European options on energy spots with spikes
- Approximate analytical pricing of European options on energy spots by the method of eigenclaims
- Analytical pricing of European call and put options on energy spots with Pareto spikes.

# Session 5

#### Pricing and Hedging Single-Commodity European Options on Energy Forwards and Swaps

- Pricing and hedging European options on energy forwards and swaps for energy spots with spikes in the non-Markovian approach
- Approximate analytical pricing and hedging of European energy options on energy forwards and swaps by the methods of eigenclaims
- Approximate pricing and dynamic hedging of European options on energy forwards and swaps for energy spots with spikes by the method of quasidifferential operators in the framework of the beliefs-preferences gauge symmetry
- Analytical pricing and hedging of European call and put options on energy forwards and swaps for energy spots with Pareto spikes
- Analytical modeling of the implied volatility for European call and put options on energy forwards and swaps for energy spots with Pareto spikes

# Day 2

# Session 6

#### Pricing and Hedging Multi-Commodity European Options on Energy Spots

- Pricing European options on several energy spots with or without spikes in the non-Markovian approach
- Linear evolution equation for European options on several energy spots with or without spikes
- Modeling correlations of energy spots with or without spikes
- Approximate analytical pricing of European options on several energy spots with or without spikes by the method of eigenclaims
- Pricing European call and put spread options on energy spots with or without spikes
- Examples of the transportation, spark spread and transmission options on spots

# Session 7

#### Pricing and Hedging Multi-Commodity European Options on Energy Forwards and Swaps

- Pricing and hedging European options on several energy forwards and swaps for energy spots with or without spikes
- Modeling correlations of energy forwards and swaps for energy spots with or without spikes
- Approximate analytical pricing and hedging of European energy options on energy forwards and swaps for energy spots with or without spikes by the method of eigenclaims
- Approximate pricing and dynamic hedging of European options on several energy forwards and swaps for energy spots with or without spikes by the method of quasidifferential operators in the framework of the beliefs-preferences gauge symmetry
- Pricing and hedging European call and put spread options on energy forwards and swaps for energy spots with or without spikes
- Examples of the transportation, spark spread and transmission options on forwards and swaps

# Session 8

### **Pricing and Hedging Full Requirements Contracts**

- Modeling a full requirements contract as a real option
- Analytical pricing and hedging of a full requirements contract by the method of eigenclaims in the non-Markovian approach
- Analytical expression for the fixed price at which power is offered to a customer under the full requirement contract

# Session 9

# Pricing and Hedging American Energy Options

- Pricing American options on energy spots, forwards and swaps for energy spots with spikes in the non-Markovian approach:
  - the semilinear evolution equation for American options, and
  - the multi-layered trees
- Analytical subsolutions and suppersolutions of the semilinear evolution equation for American options by the method of successive approximations
- Pricing NYMEX American call and put options on crude oil and extracting related forward-looking market-implied risk-neutral probability distributions