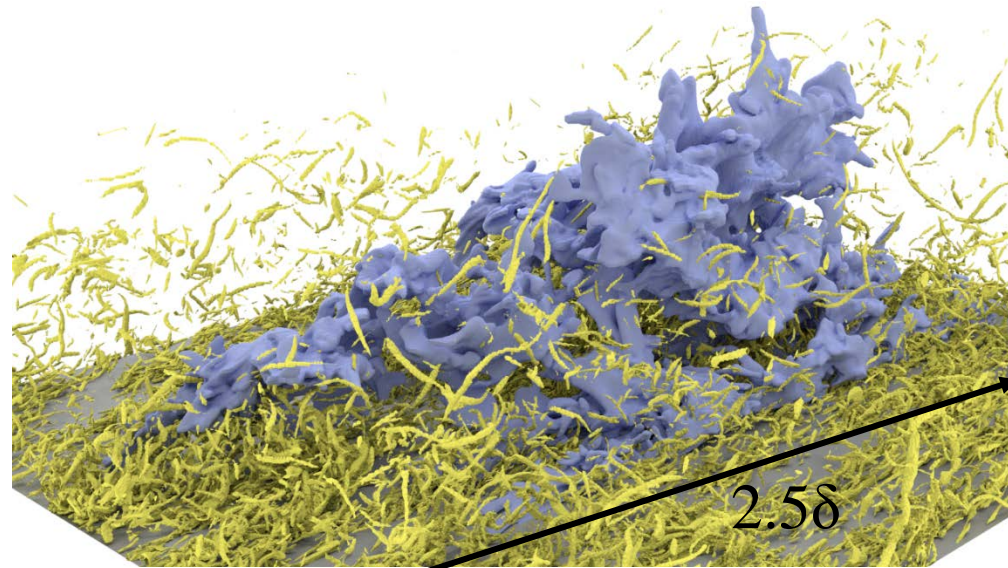


(So called) inertial eddies in turbulence

Javier Jiménez & al.

School of Aeronautics, Madrid



**TBL: $Re_\tau=1800$, $u'^+=2$
J.A. Sillero**



‘Cascades’

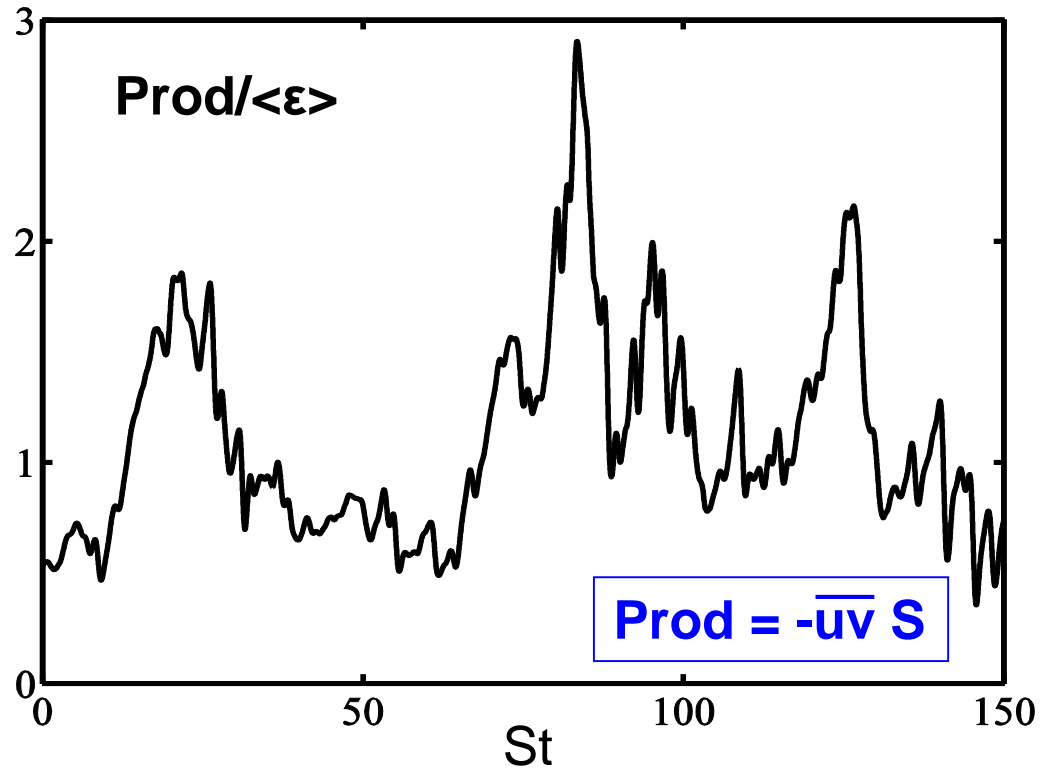
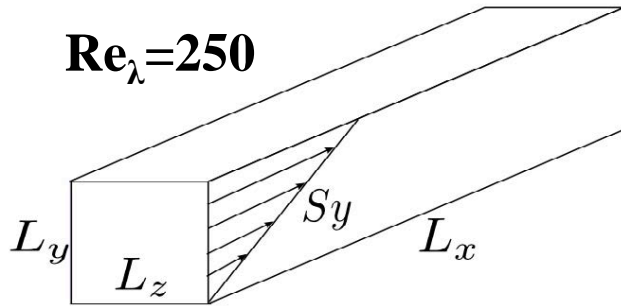
The **flux** of a conserved quantity across a range of scales

e.g.

- **Energy in Homog. Turbulence**
- **Momentum in shear flows**

The energy cascade

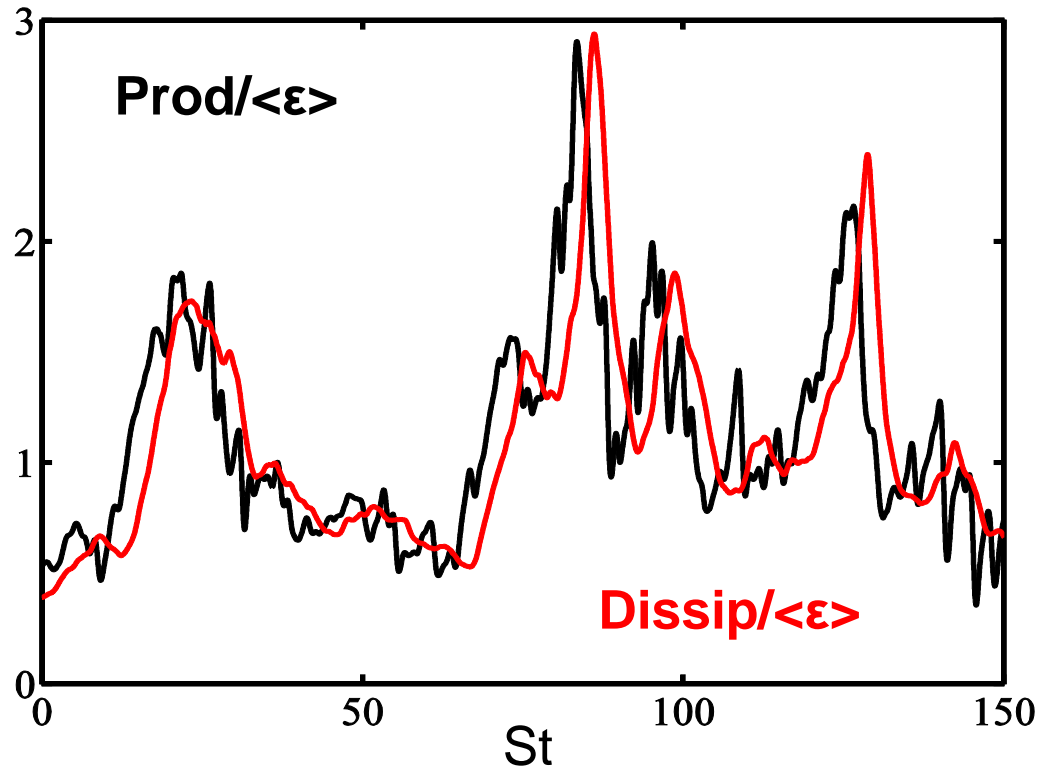
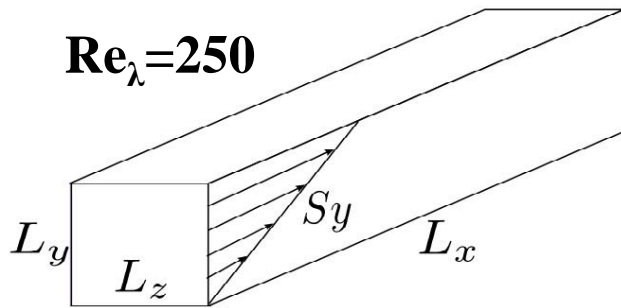
Homogeneous Shear Flow



The energy cascade takes time

Large to Small

Homogeneous Shear Flow

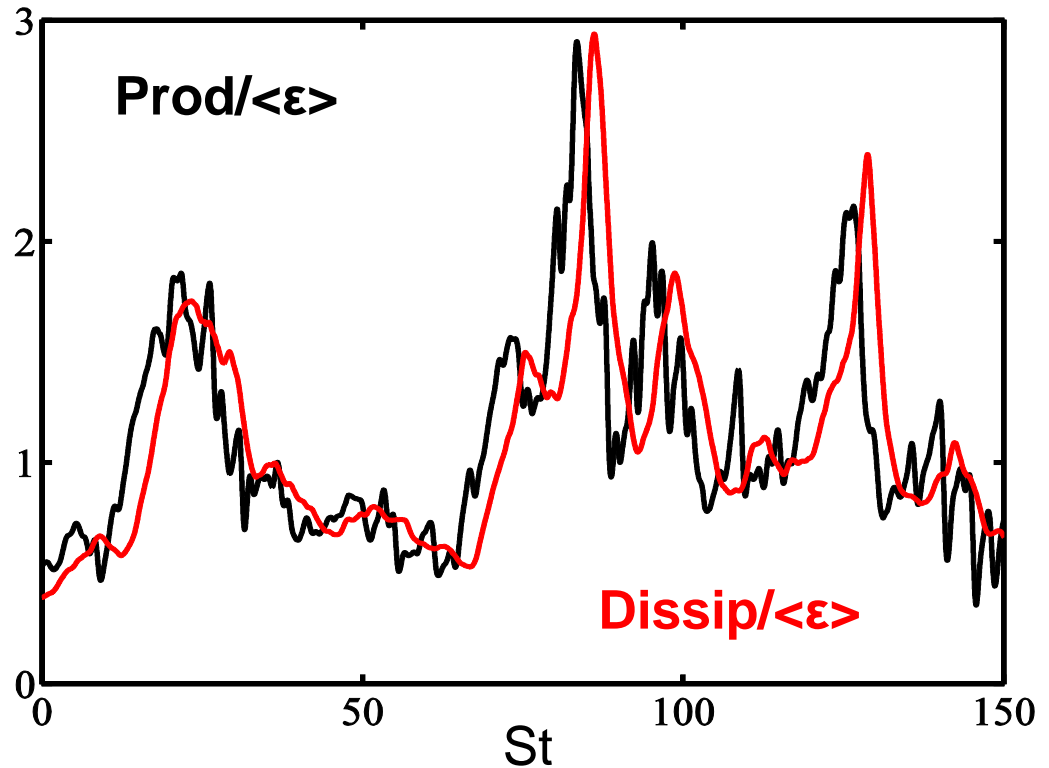
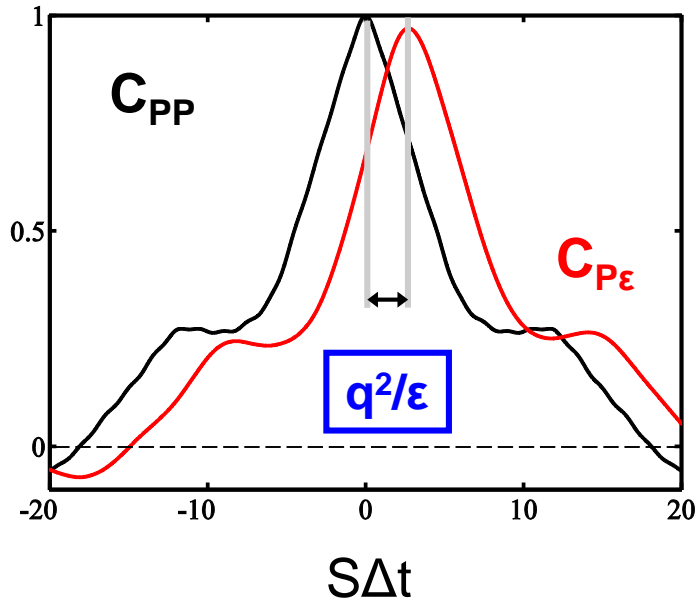


The energy cascade takes time

Large to Small

Homogeneous Shear Flow

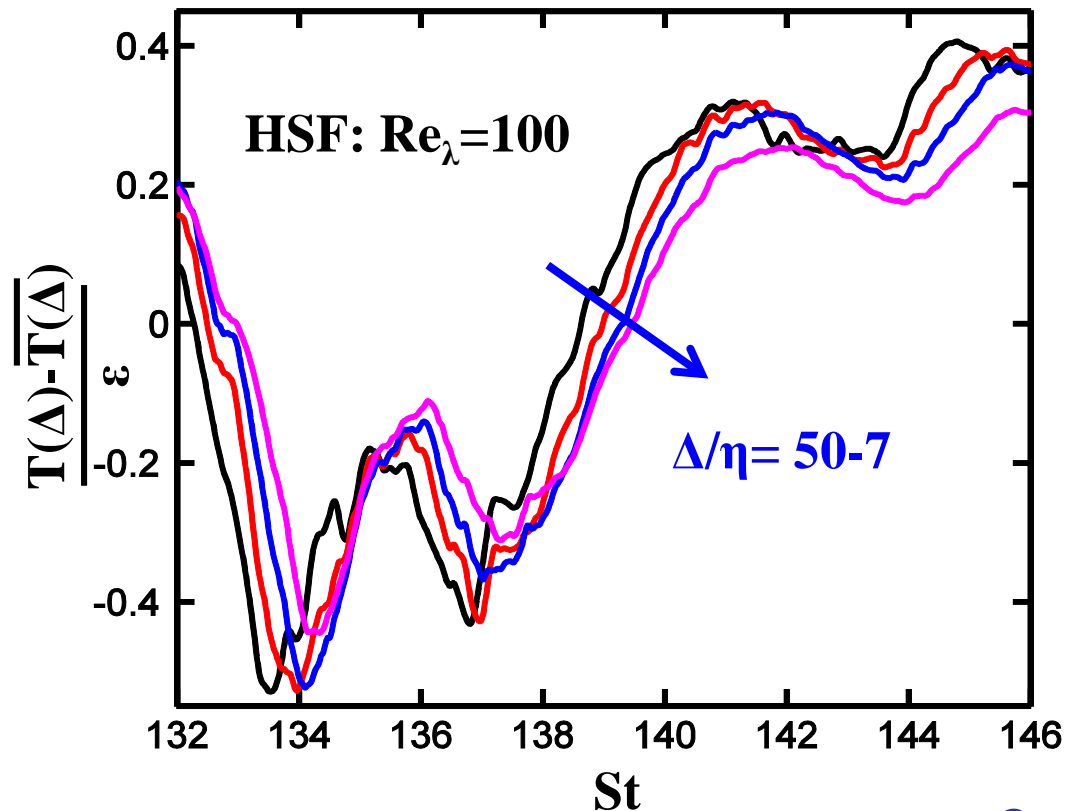
Temporal cross-correl.



The cross-scale energy flux

$$u = \bar{u} + u'; \quad \bar{u} = G*u; \quad G \sim \exp[-(x_1^2 + x_2^2 + x_3^2)/\Delta^2]$$

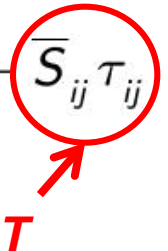
$$(\partial_t + \bar{u}_j \partial_j) \frac{1}{2} \bar{u}_i \bar{u}_i = -\partial_j (\bar{u}_j \bar{p} + \bar{u}_i \tau_{ij} - 2\nu \bar{u}_i \bar{S}_{ij}) - 2\nu \bar{S}_{ij} \bar{S}_{ij} + \bar{S}_{ij} \tau_{ij}$$



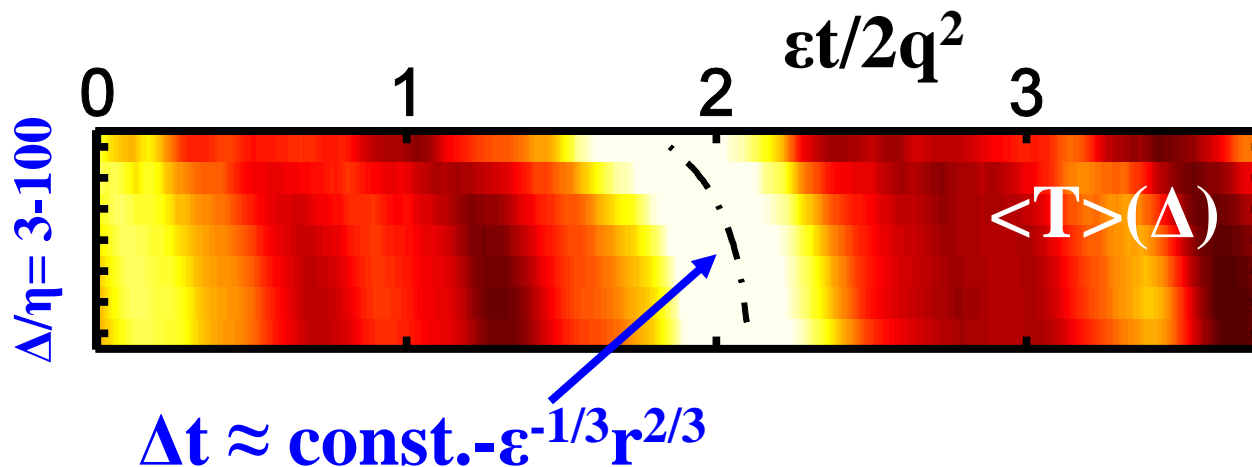
The cross-scale energy flux

$$u = \bar{u} + u'; \quad \bar{u} = G * u; \quad G \sim \exp[-(x_1^2 + x_2^2 + x_3^2) / \Delta^2]$$

$$(\partial_t + \bar{u}_j \partial_j) \frac{1}{2} \bar{u}_i \bar{u}_i = -\partial_j \left(\bar{u}_j \bar{p} + \bar{u}_i \tau_{ij} - 2\nu \bar{u}_i \bar{S}_{ij} \right) - 2\nu \bar{S}_{ij} \bar{S}_{ij} + \bar{S}_{ij} \tau_{ij}$$

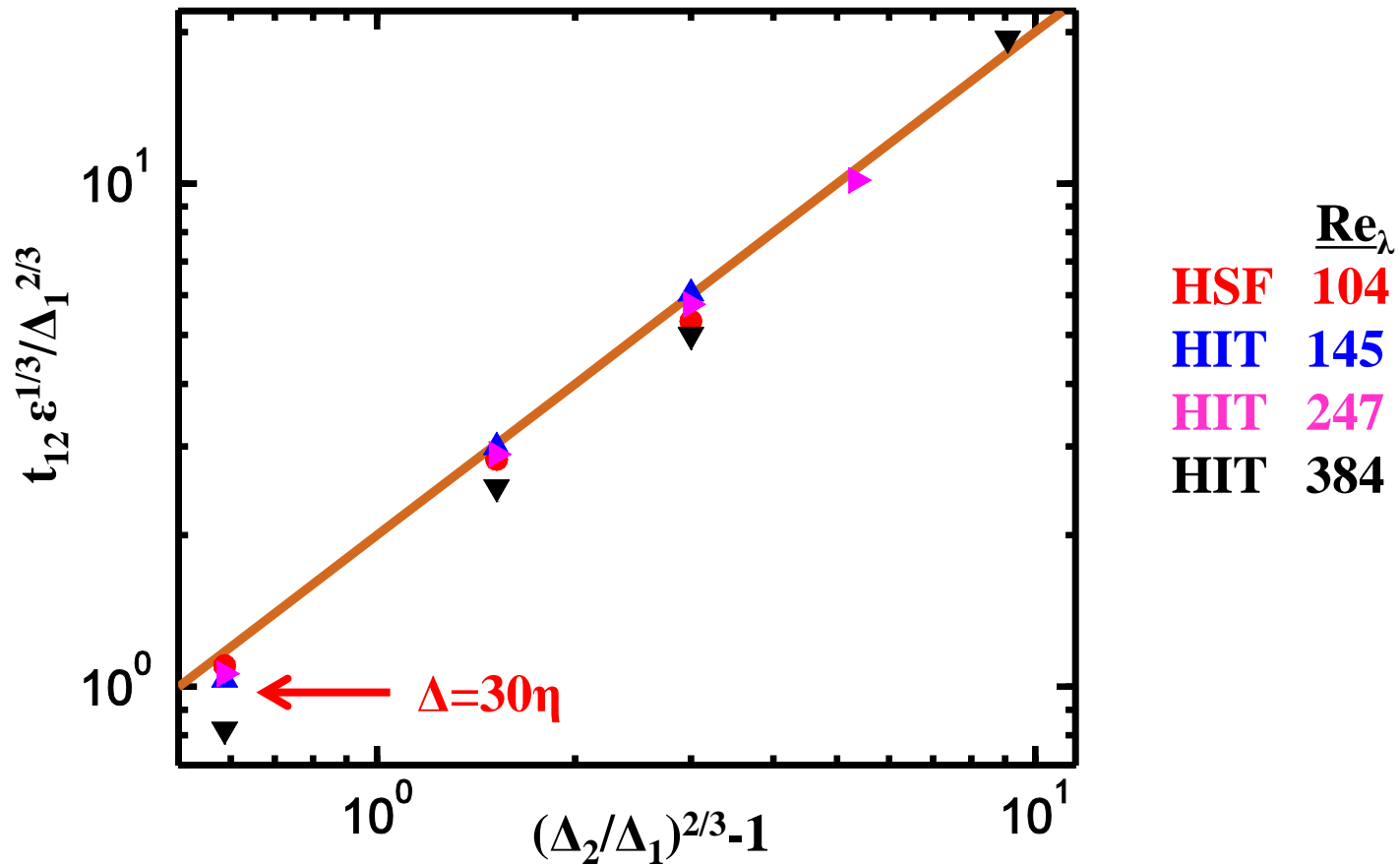


 T



HIT: $Re_\lambda = 384$

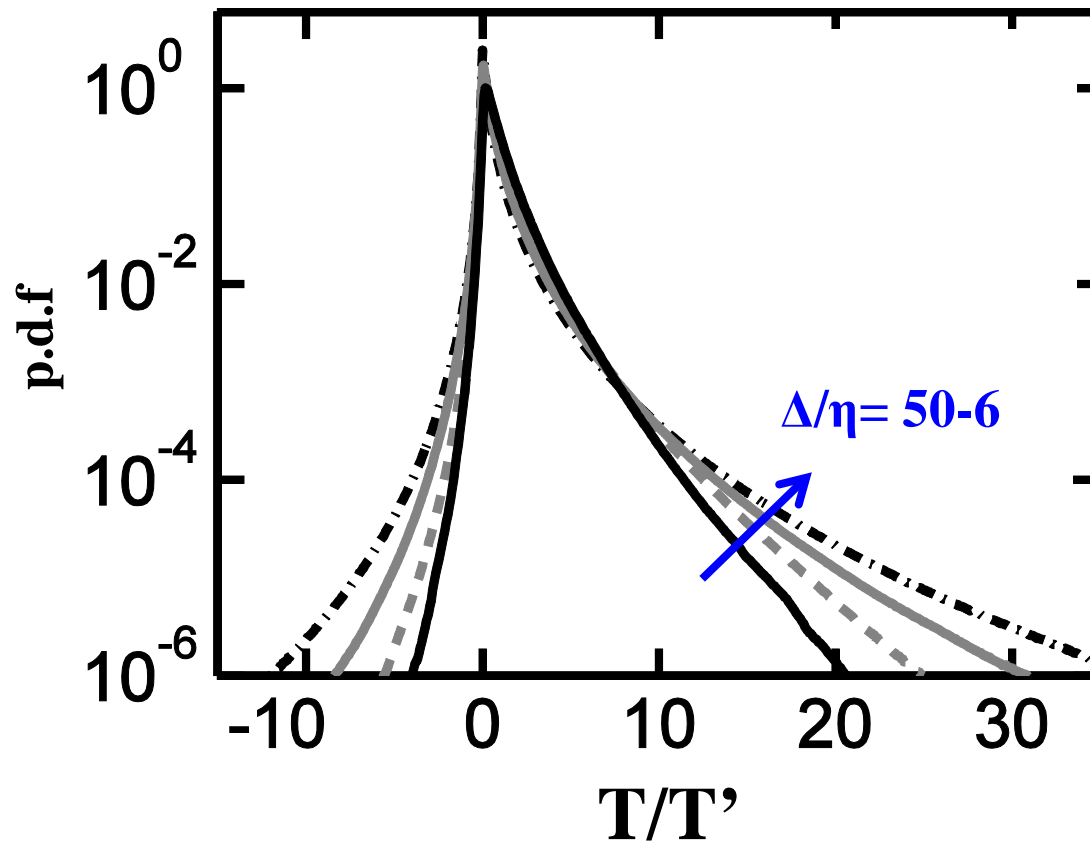
The 'velocity' of the cascade



The energy flux is intermittent

$$u = \bar{u} + u'; \quad \bar{u} = G * u; \quad G \sim \exp[-(x_1^2 + x_2^2 + x_3^2) / \Delta^2]$$

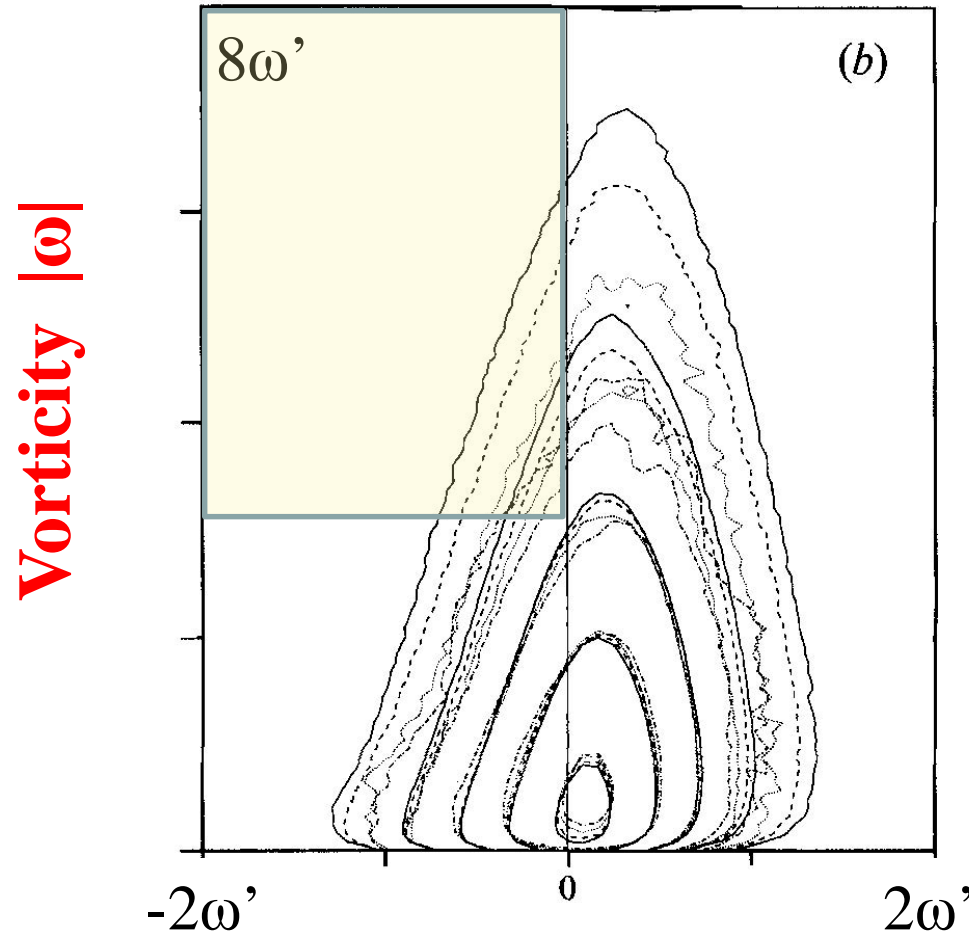
$$(\partial_t + \bar{u}_j \partial_j) \frac{1}{2} \bar{u}_i \bar{u}_i = -\partial_j (\bar{u}_j \bar{p} + \bar{u}_i \tau_{ij} - 2\nu \bar{u}_i \bar{S}_{ij}) - 2\nu \bar{S}_{ij} \bar{S}_{ij} + \bar{S}_{ij} \tau_{ij}$$



$\bar{S}_{ij} \tau_{ij}$
↑
 T

HSF: $Re_\lambda = 100$

Stretching in turbulence

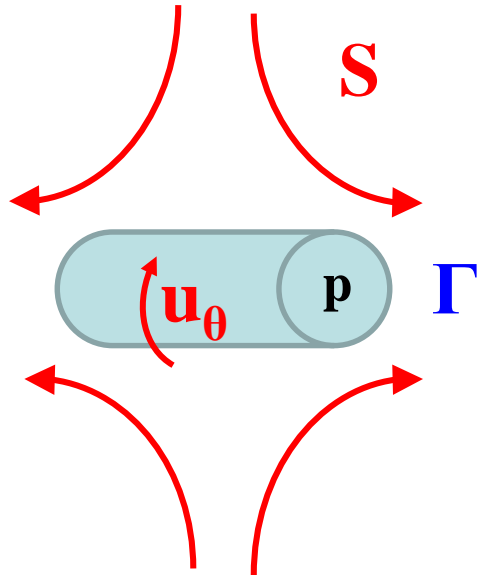


HIT, Re=60-200

Stretching
 $\omega S \omega / |\omega|^2$

Jimenez, Wray, Saffman, Rogallo (1993,98)

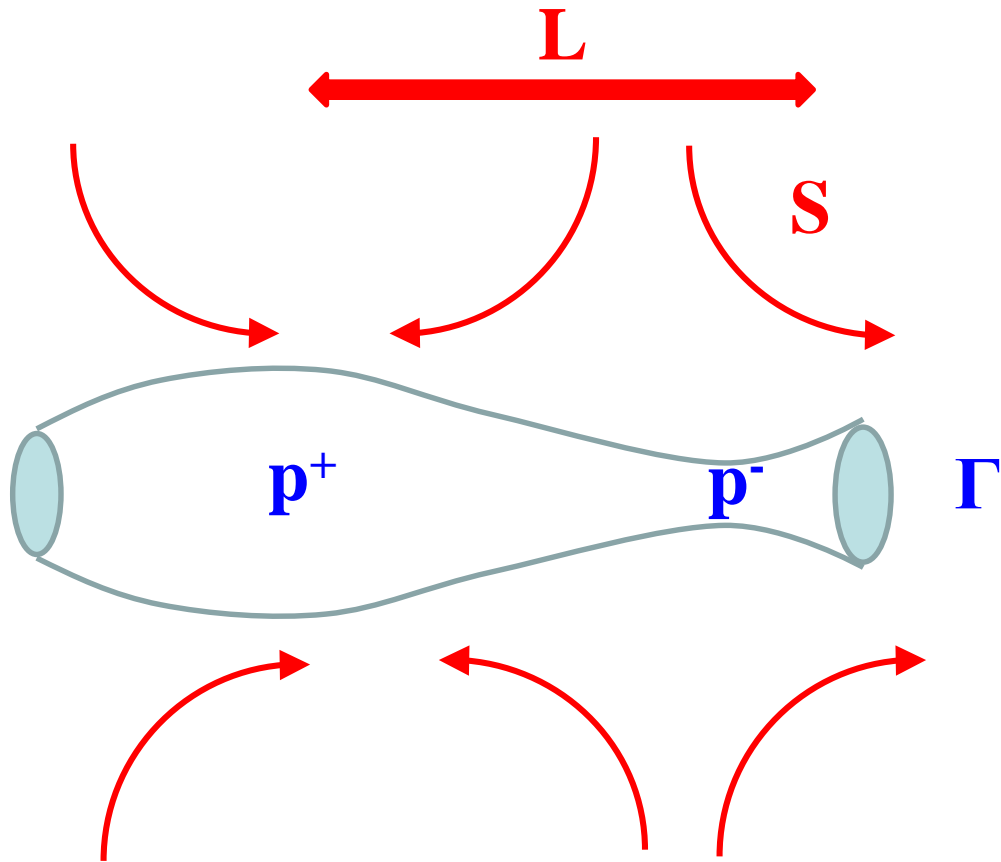
Stretching an **infinite** vortex



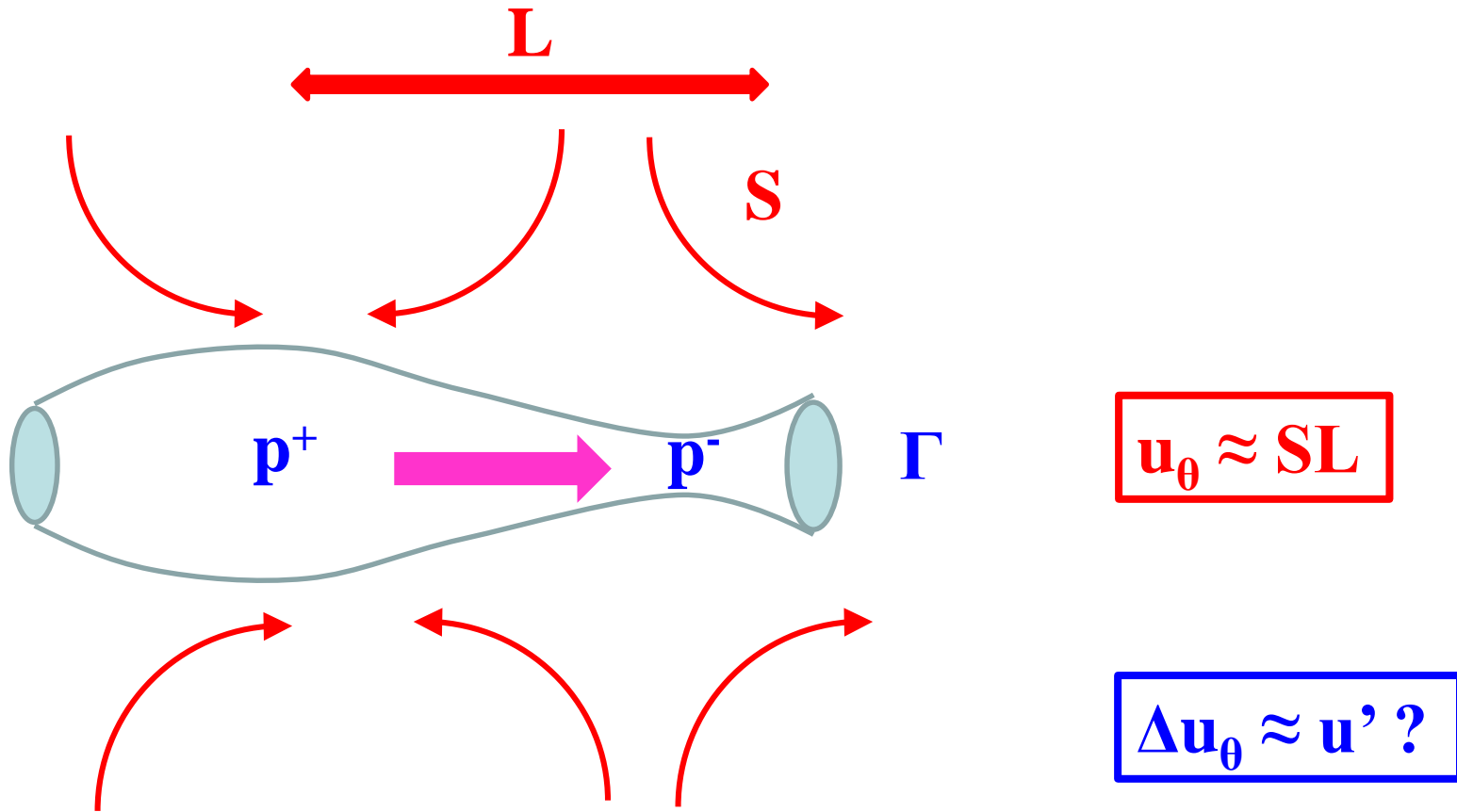
$$u_\theta = \Gamma (S/\nu)^{1/2}$$

$$p \approx -u_\theta^2$$

Stretching an **finite** vortex

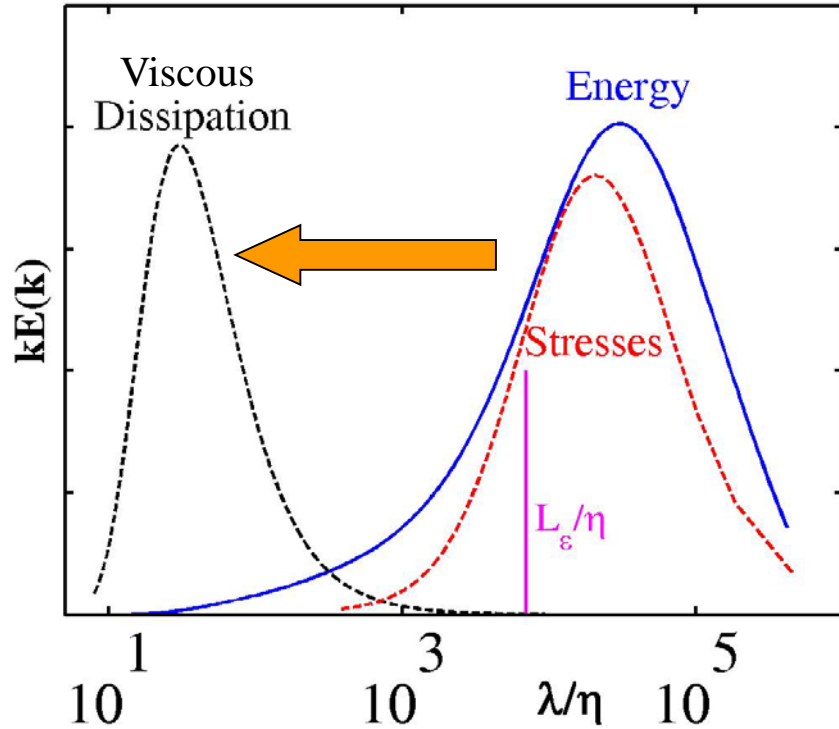


Stretching an **finite** vortex

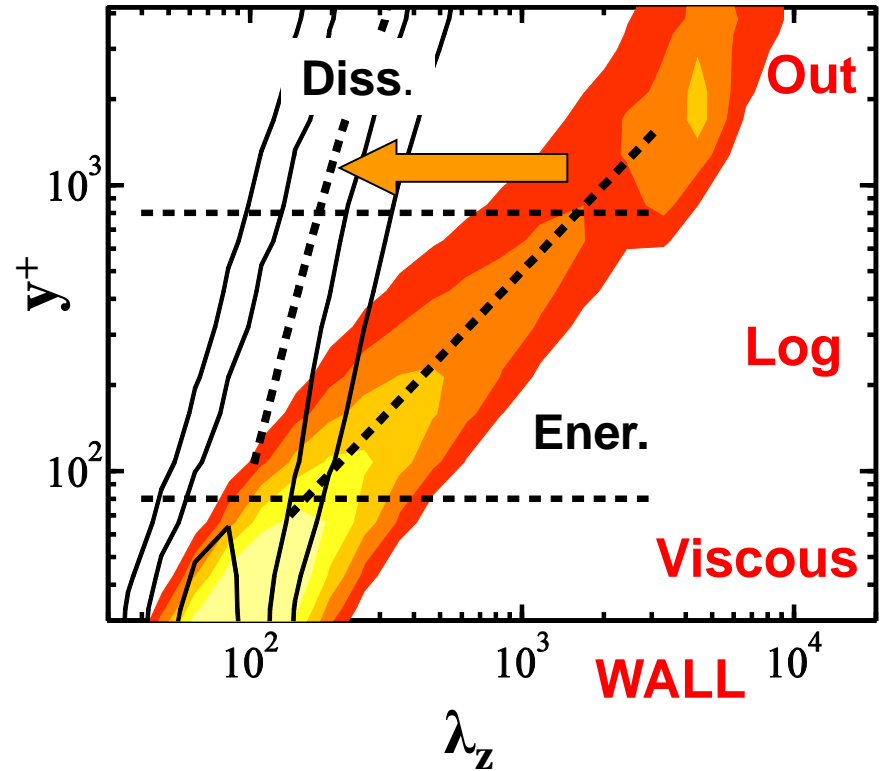


Wall-bounded Flows

Homogeneous Turbulence

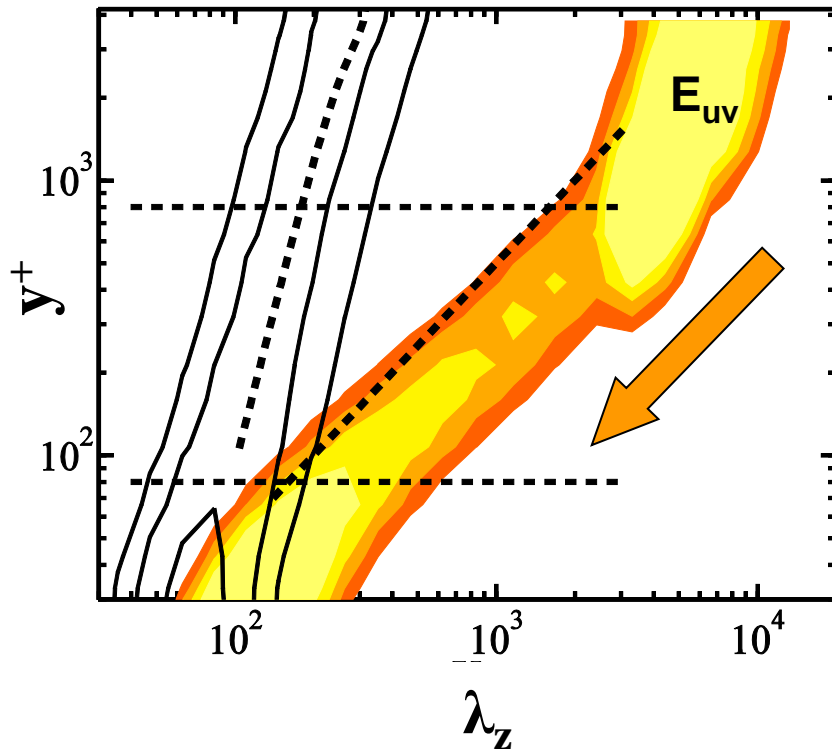


Wall Turbulence (Channel)

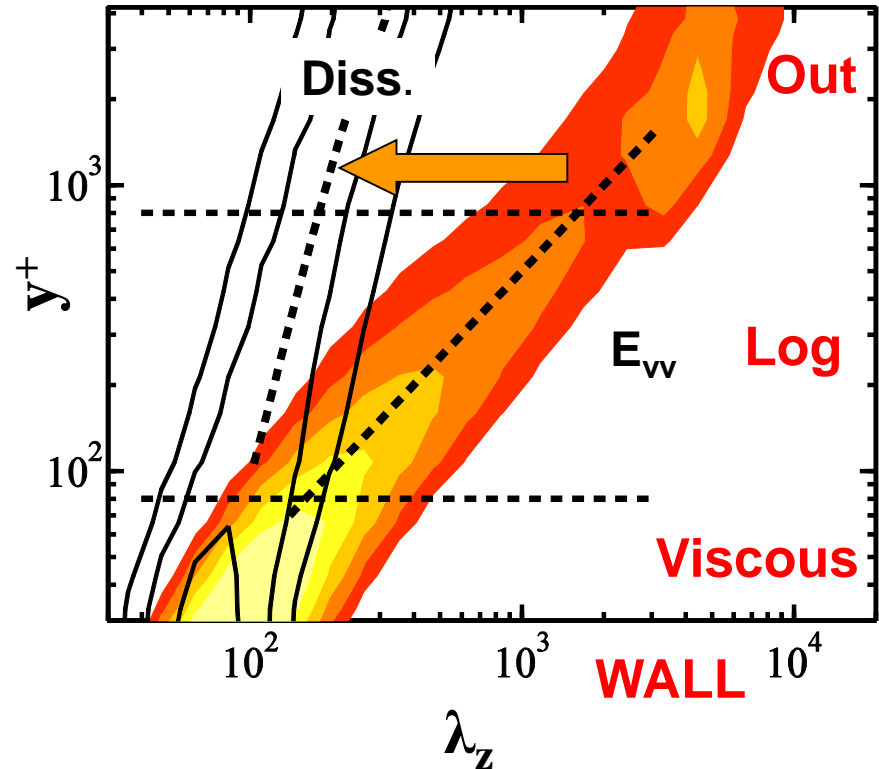


Momentum Flux Cascade

Momentum (Channel)



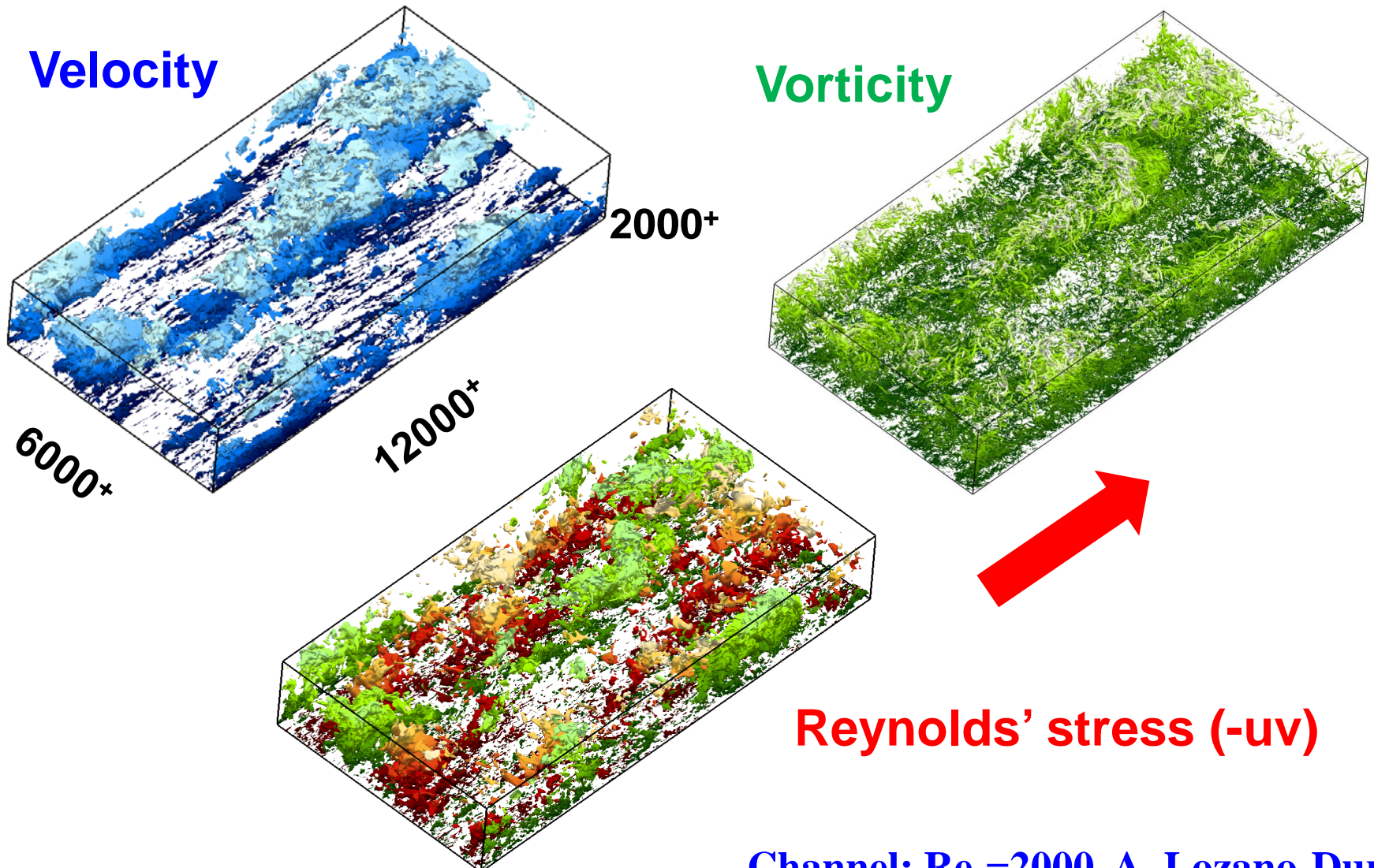
Energy (Channel)



$$\partial_t U + \partial_x P = \partial_y \langle -uv \rangle + O(\text{Re}_\tau^{-1})$$

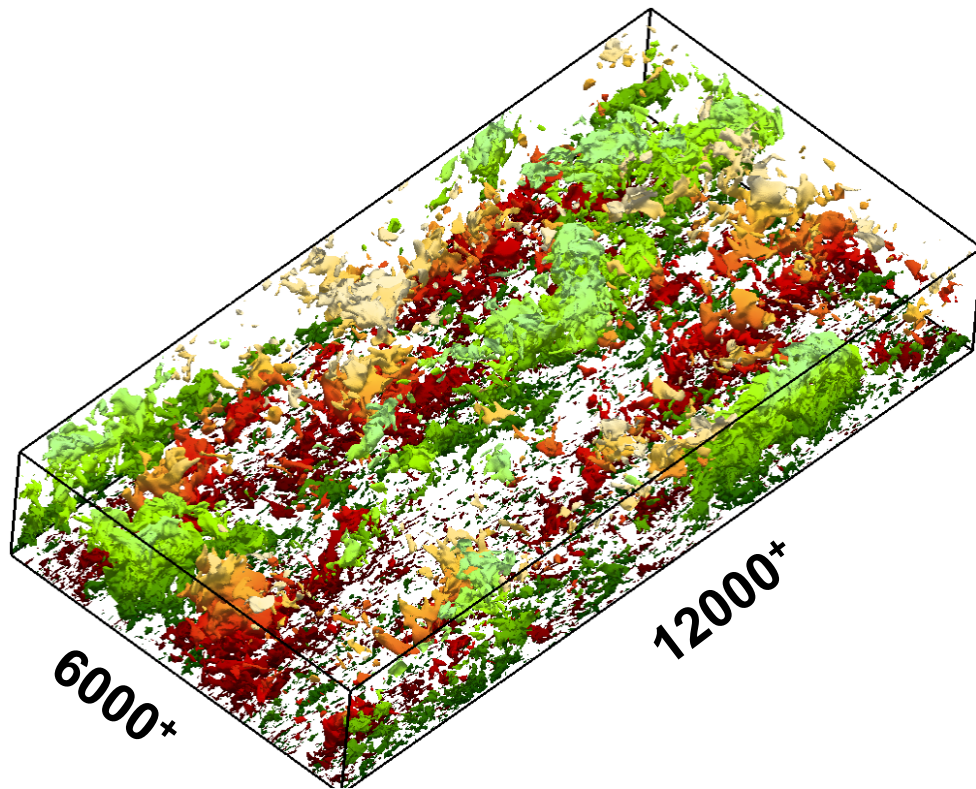
$$\tau = -uv \quad \leftarrow \text{Momentum flux}$$

Flow fields of the Logarithmic layer

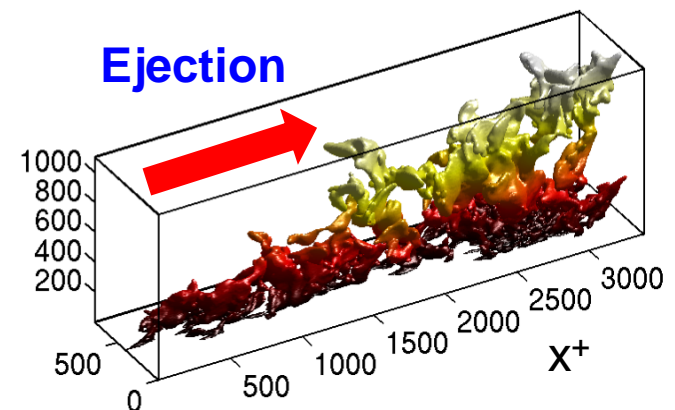
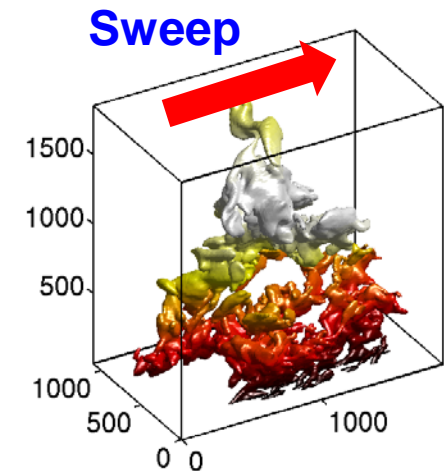


Channel: $Re_\tau=2000$. A. Lozano-Durán

Momentum Structures of the Logarithmic layer

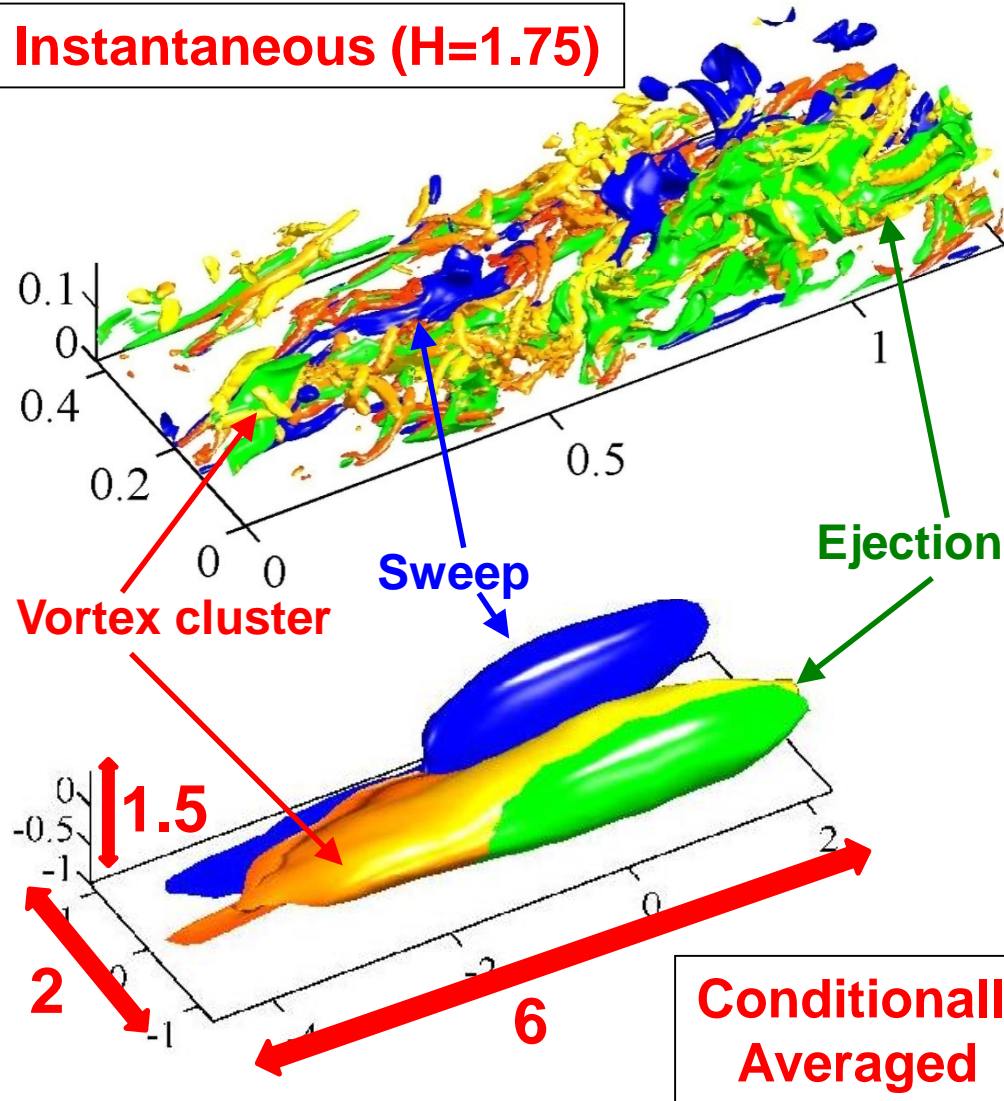


$$-uv > 1.75 u'v'$$

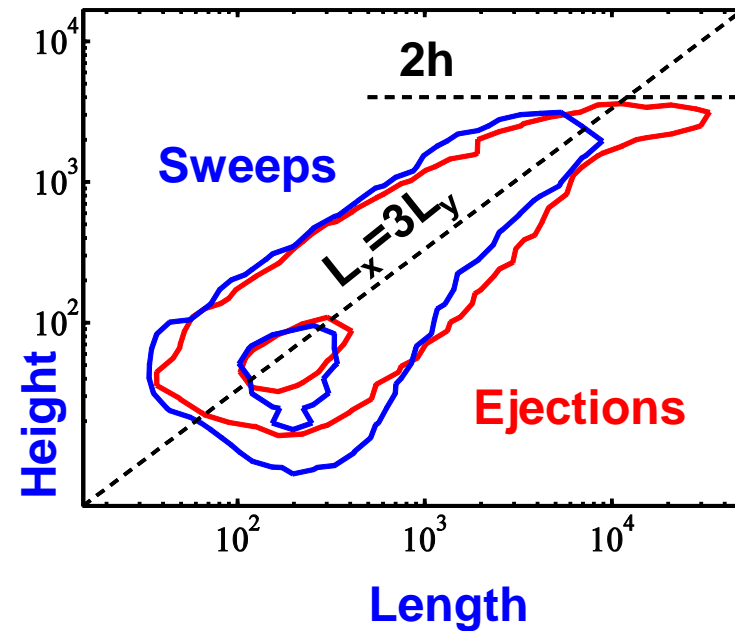


Attached Sweeps and Ejections

Instantaneous ($H=1.75$)



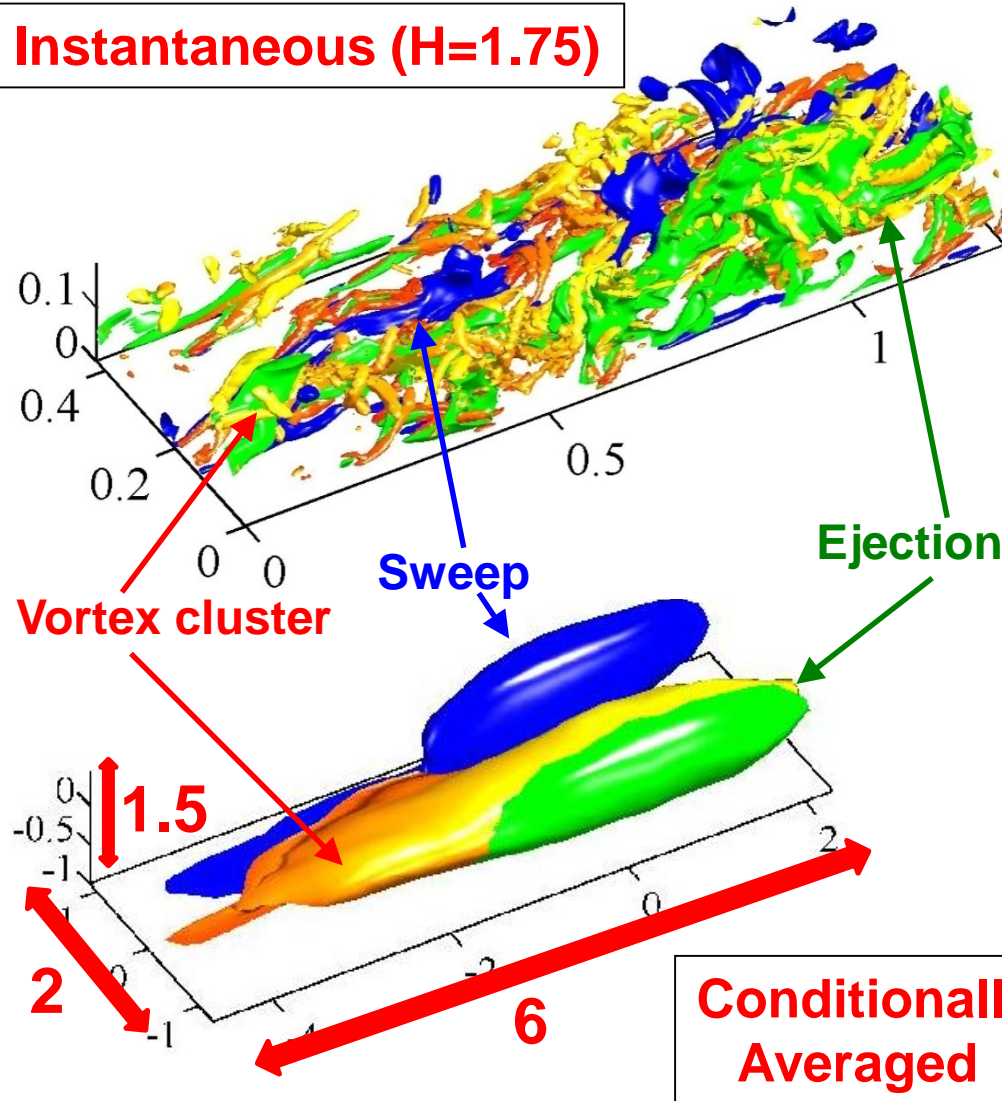
Momentum Transfer
is self-similar



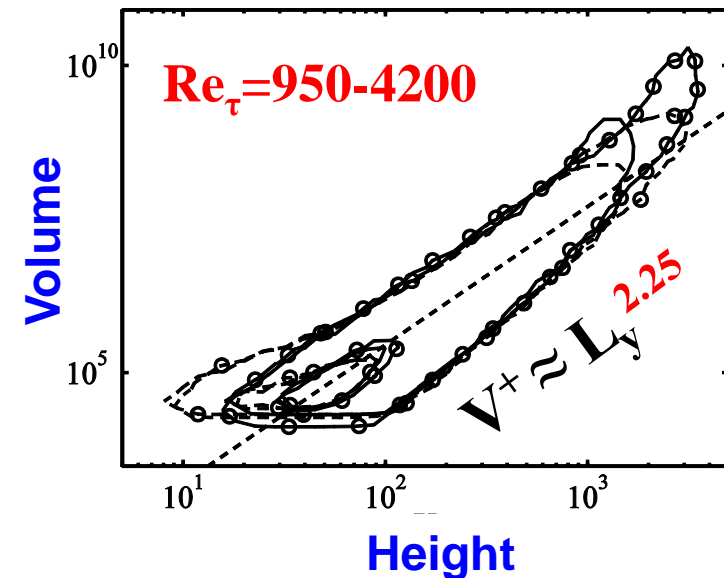
Lozano-Duran, Flores & J (2012)

Attached Sweeps and Ejections

Instantaneous ($H=1.75$)



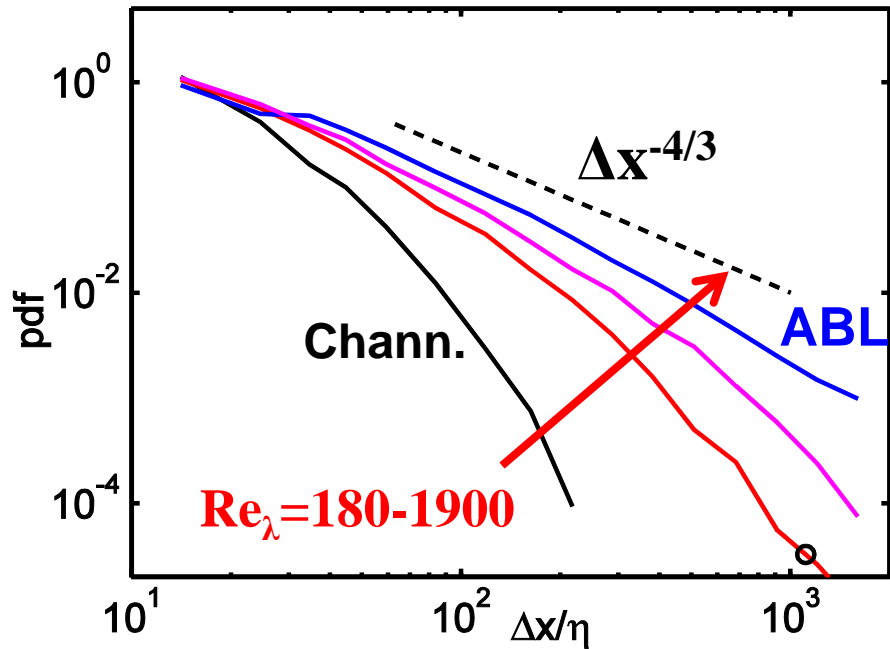
Momentum Transfer
is Fractal



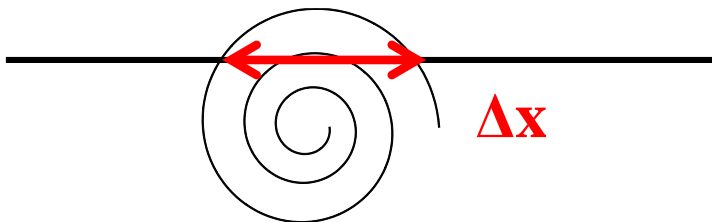
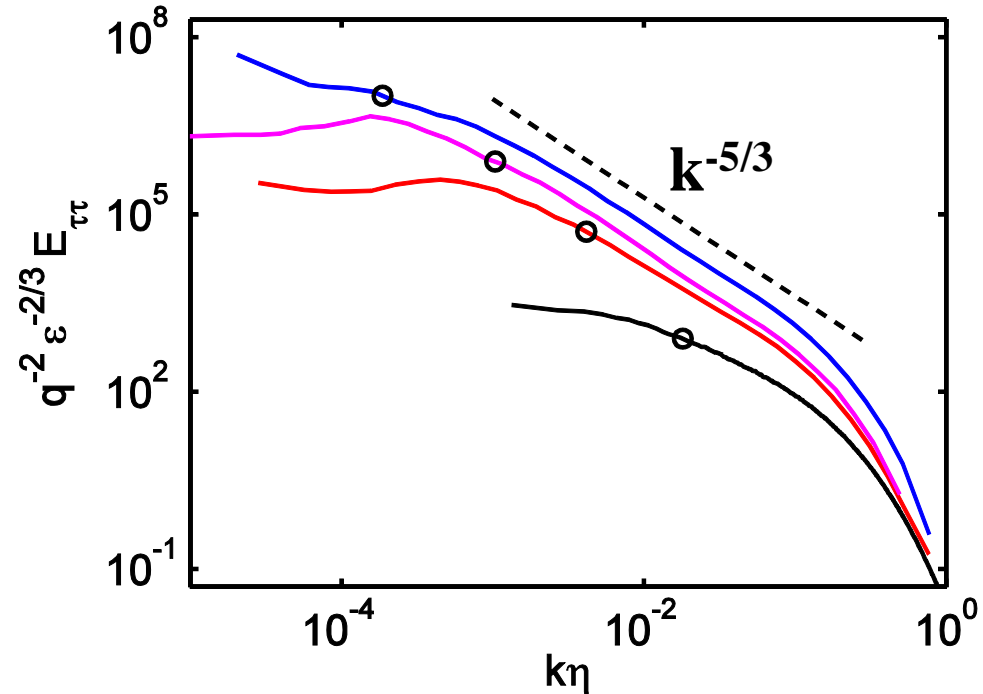
Lozano-Duran, Flores & J (2012)

Momentum Transfer is “Universal”

Length of (uv)

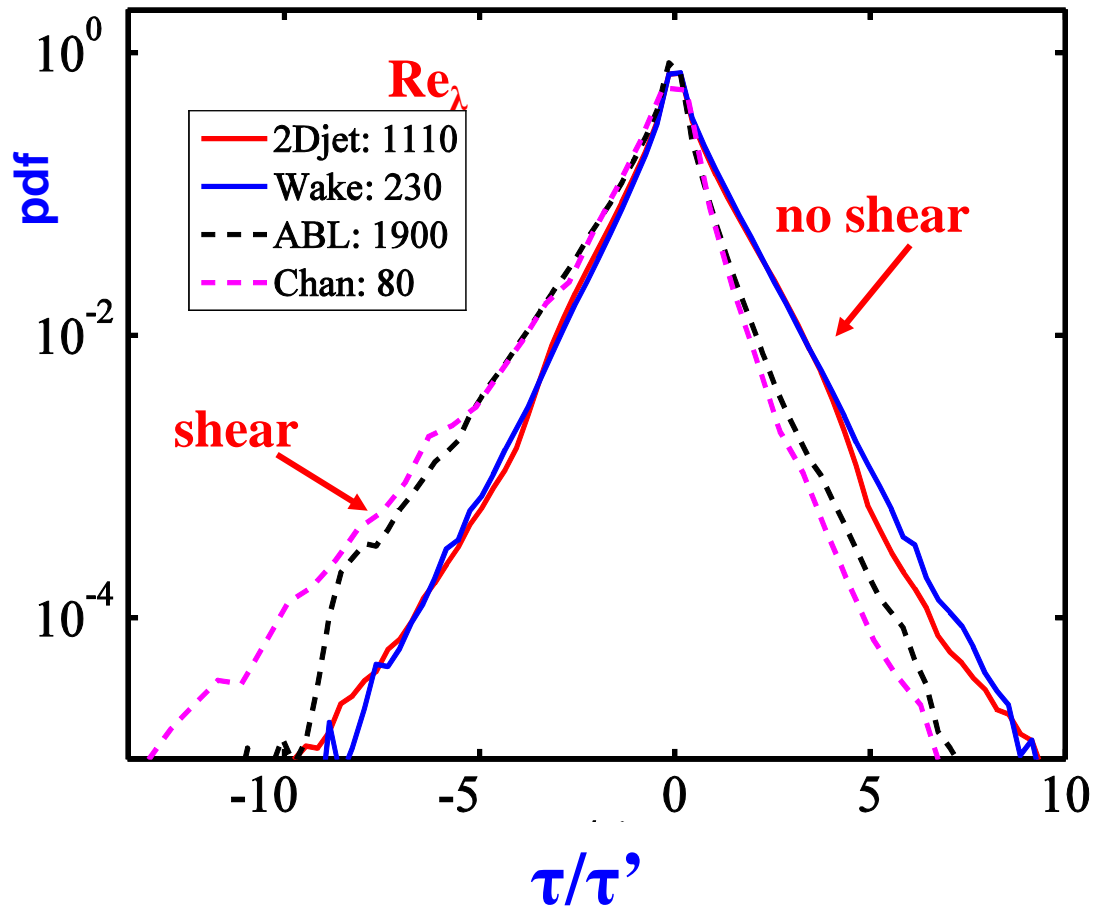


Spectrum of (uv)

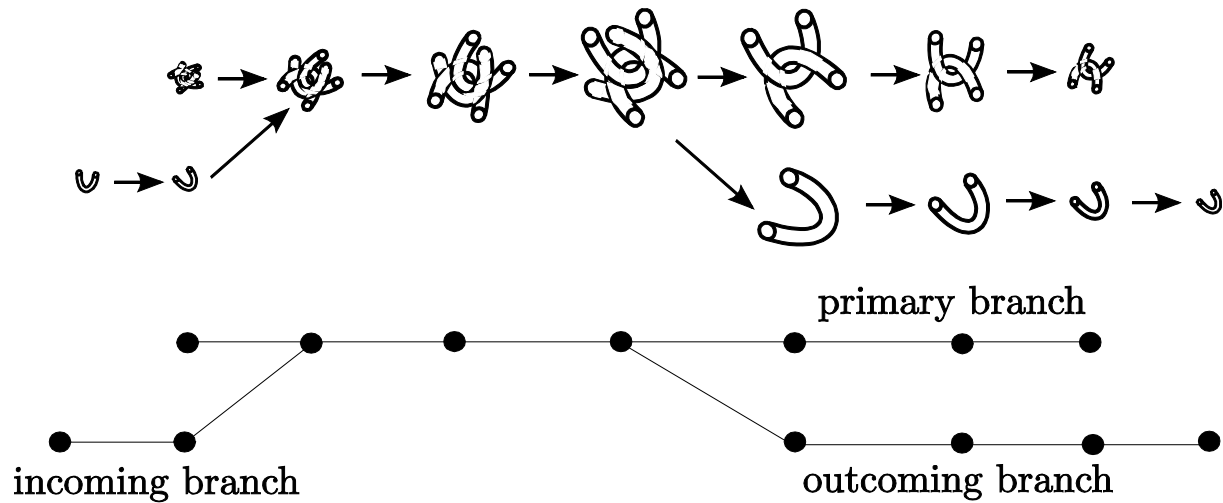
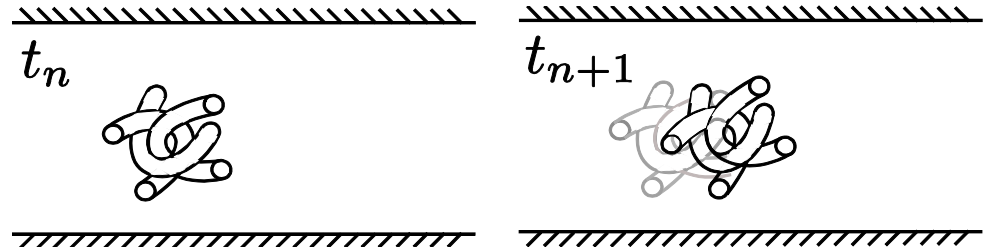
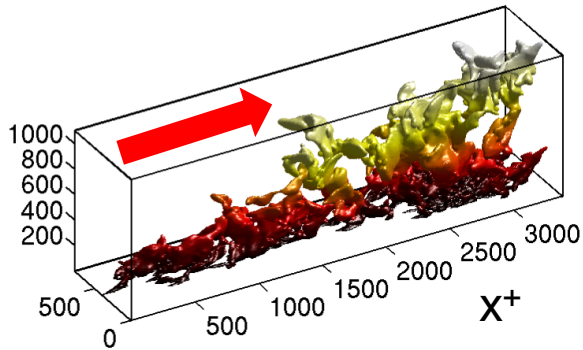


Sweeps and Ejections

Momentum Transfer is **NOT** Intermittent

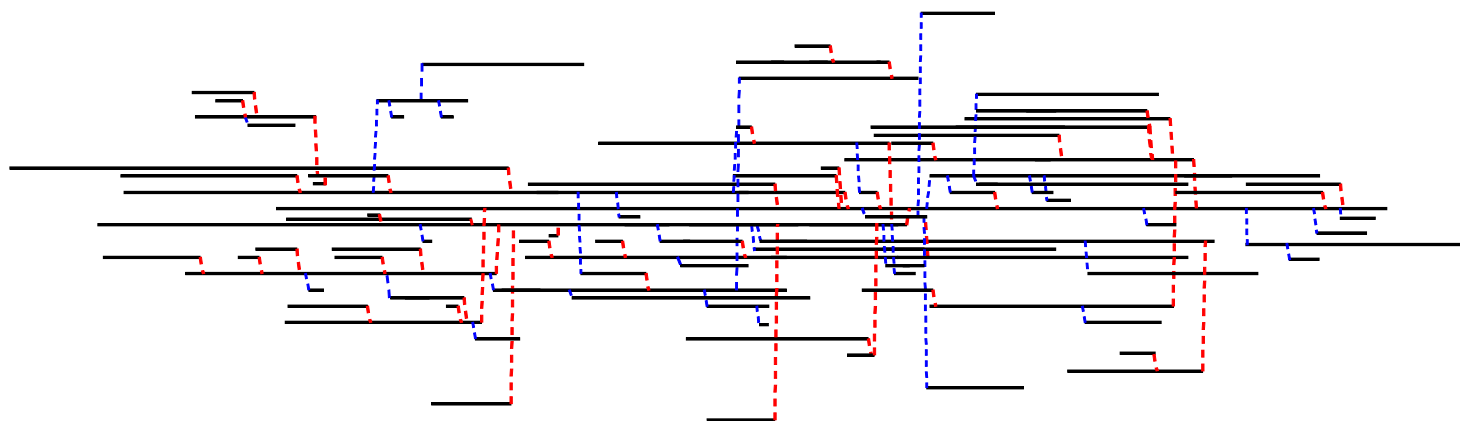
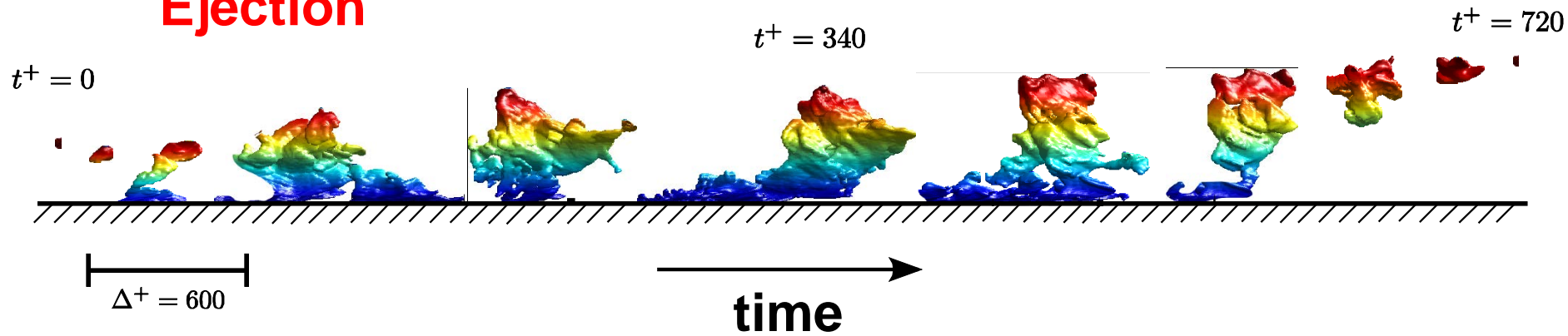


Tracking Eddies in Time



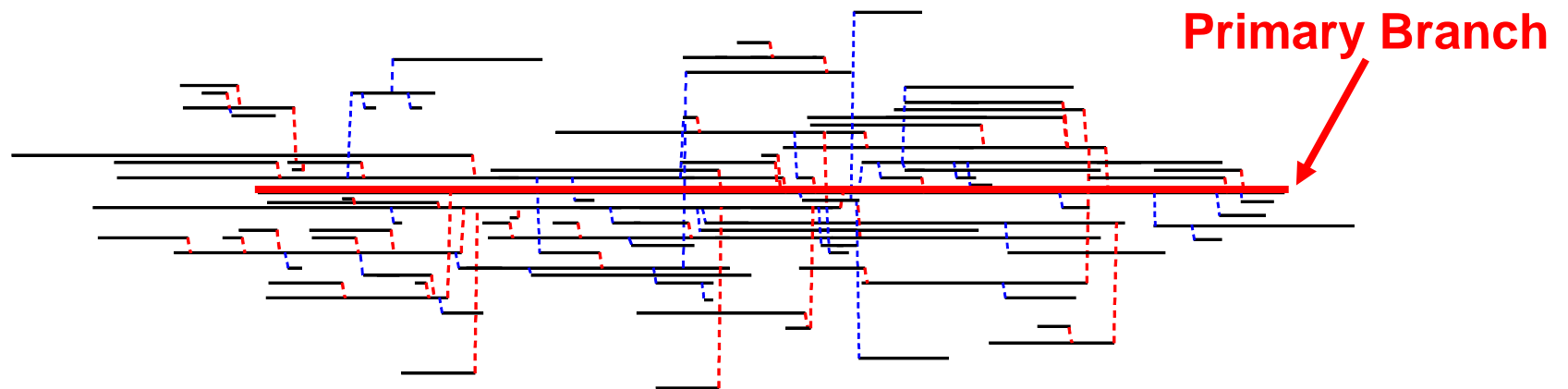
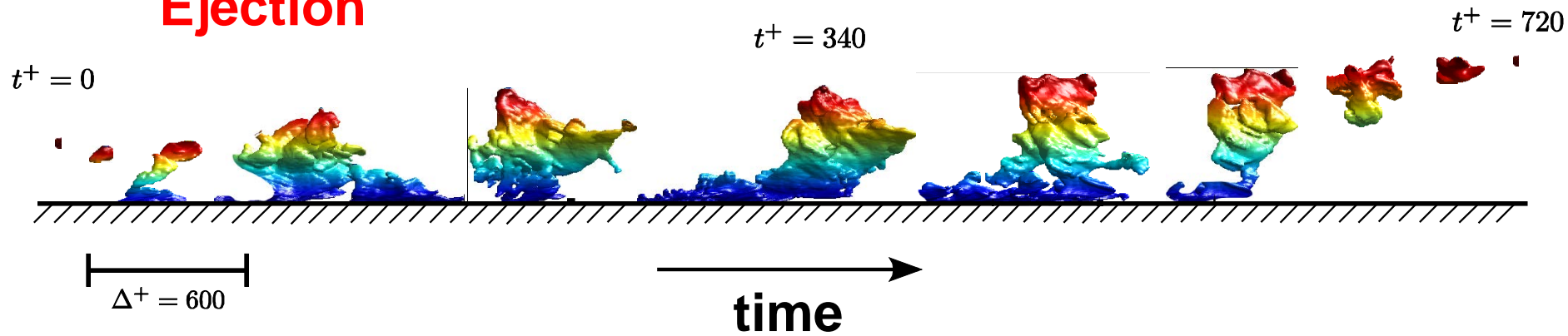
Tracking in Time

Ejection

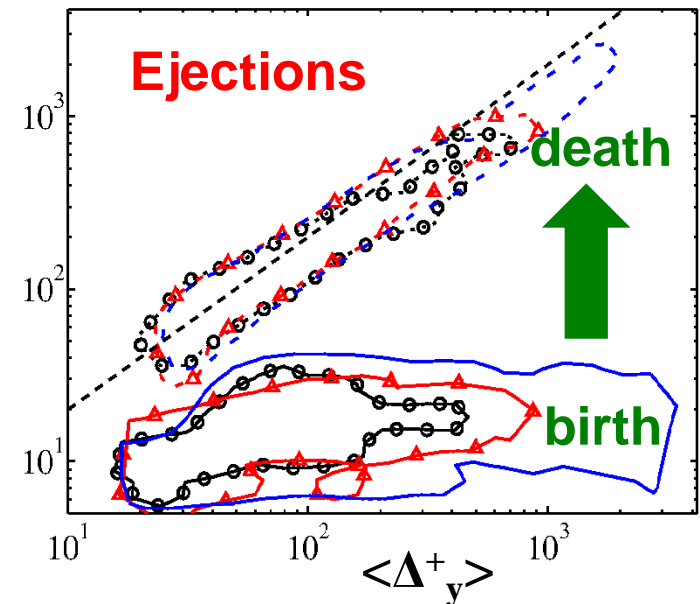
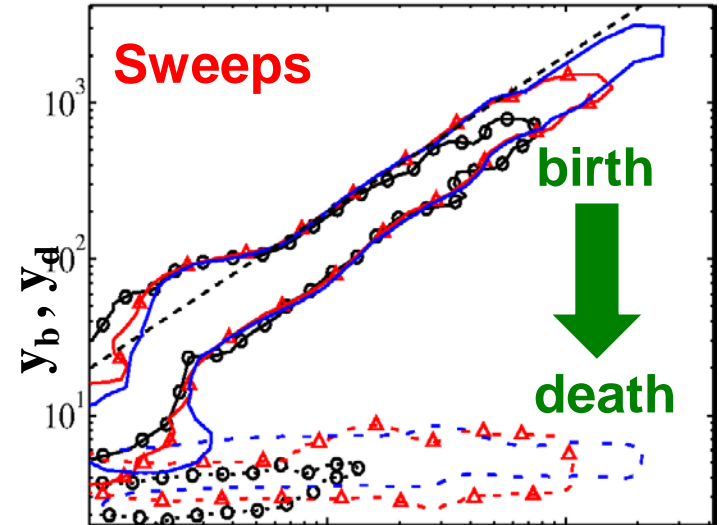
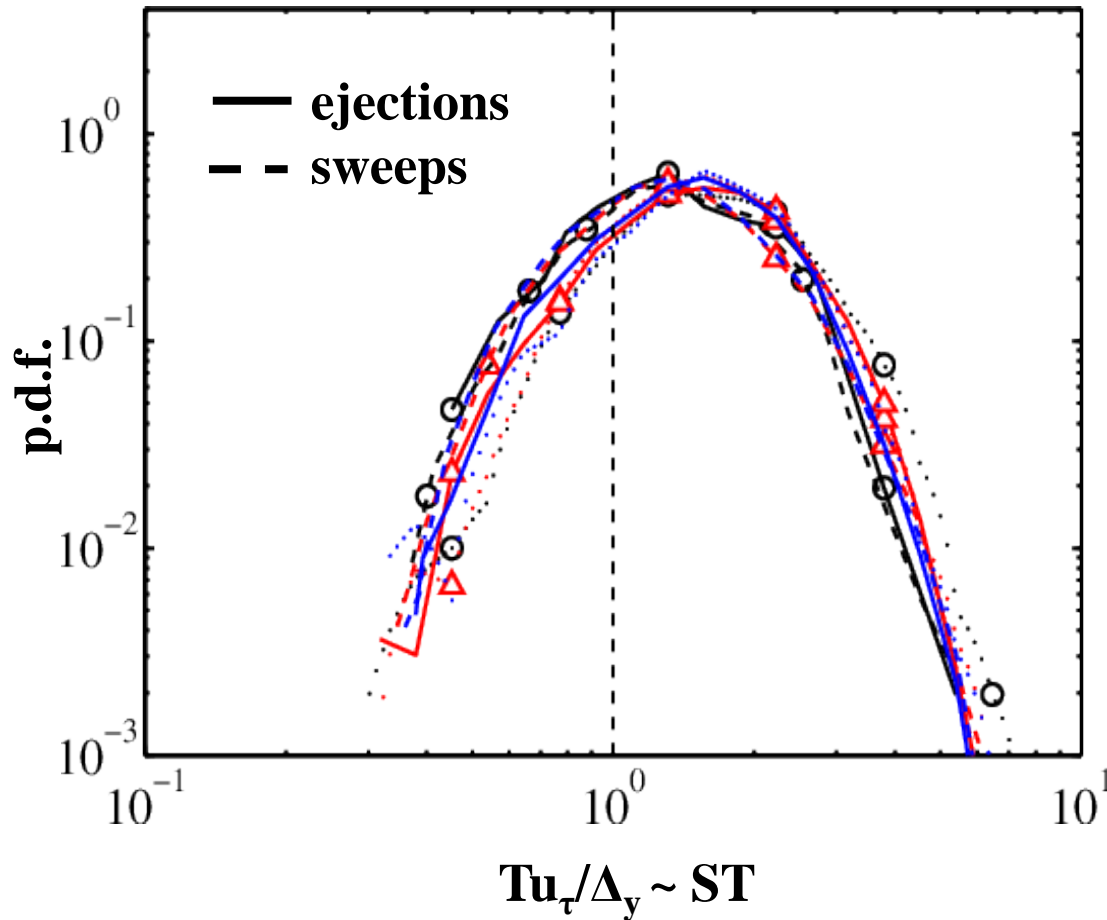


Tracking in Time

Ejection



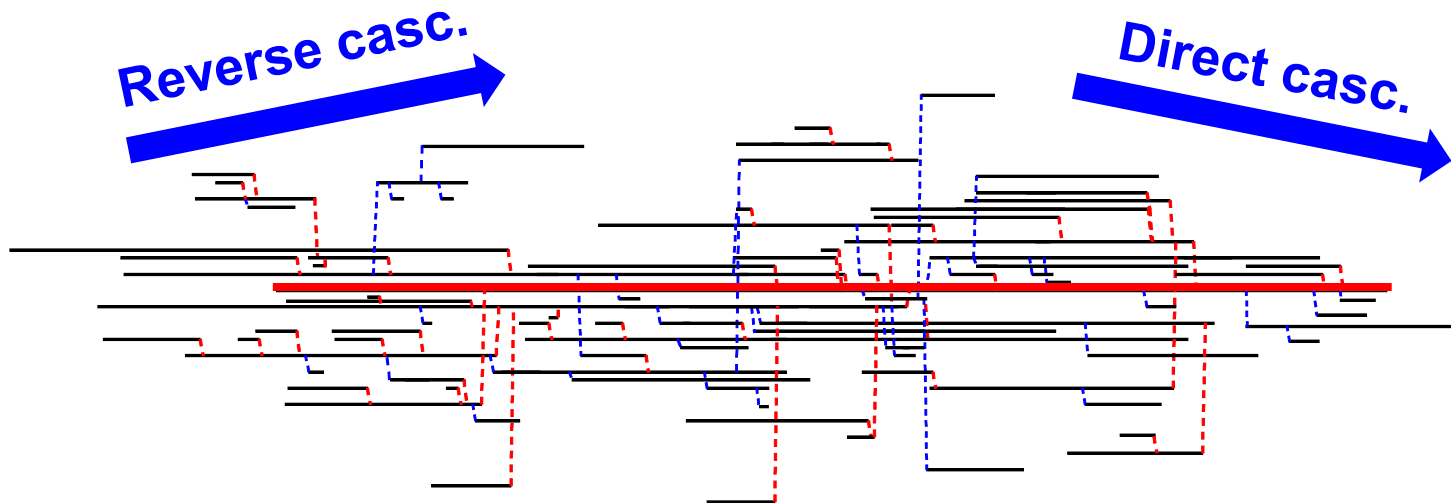
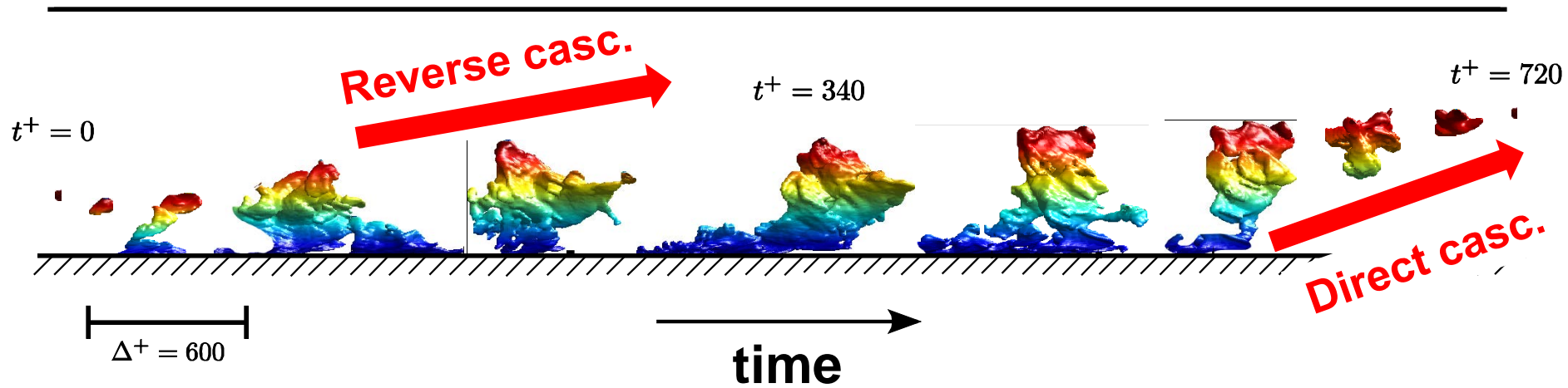
Lifetimes: **Attached** Sweeps and Ejections



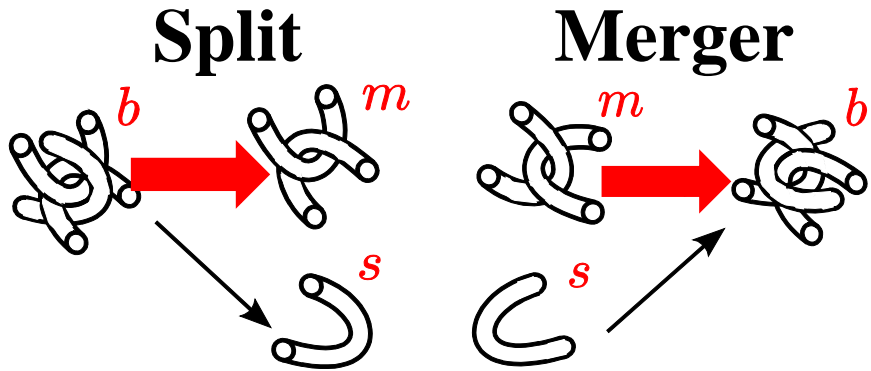
Channels: $Re_\tau = 950-4200$

Lozano-Durán & J (2014)

Scale Change in Time



Splits and Mergers



$$V_b \approx V_m + V_s$$

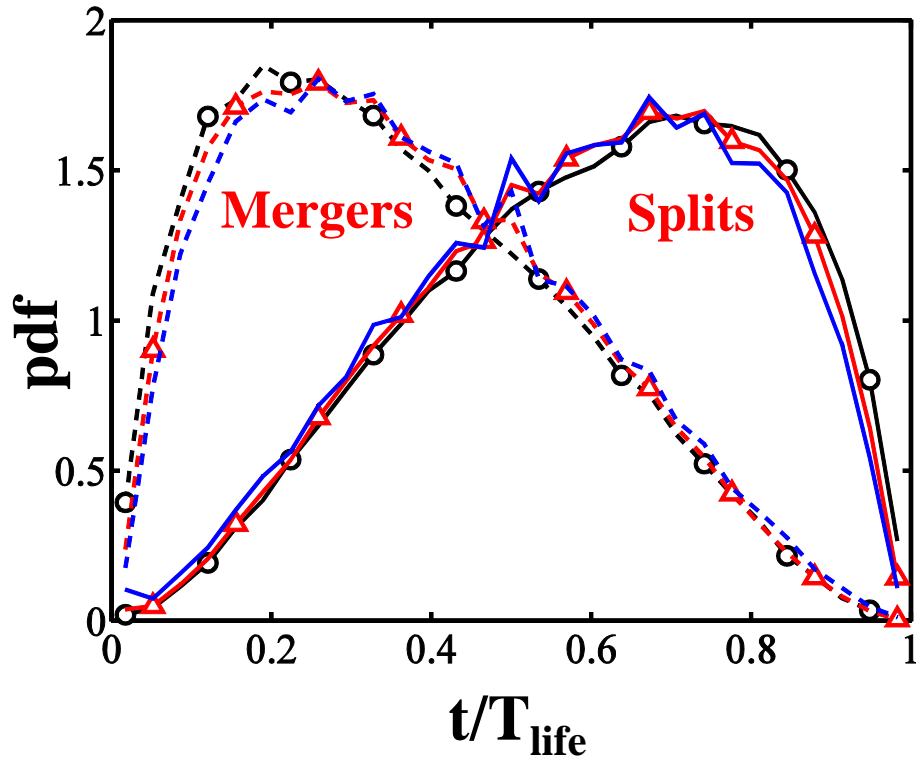
$$\Delta V_{\text{BRANCH}} = \sum |\Delta V| \approx \sum V_s$$

Δ_y/η	Inertial $\Delta_s > 100\eta$	Viscous $\Delta_s < 100\eta$	Smooth Growth
0-50	0%	23%	77%
50-200	28%	23%	49%
200-400	54%	8%	38%
>400	94%	5%	1%

$Re_\tau = 4200$; “detached”

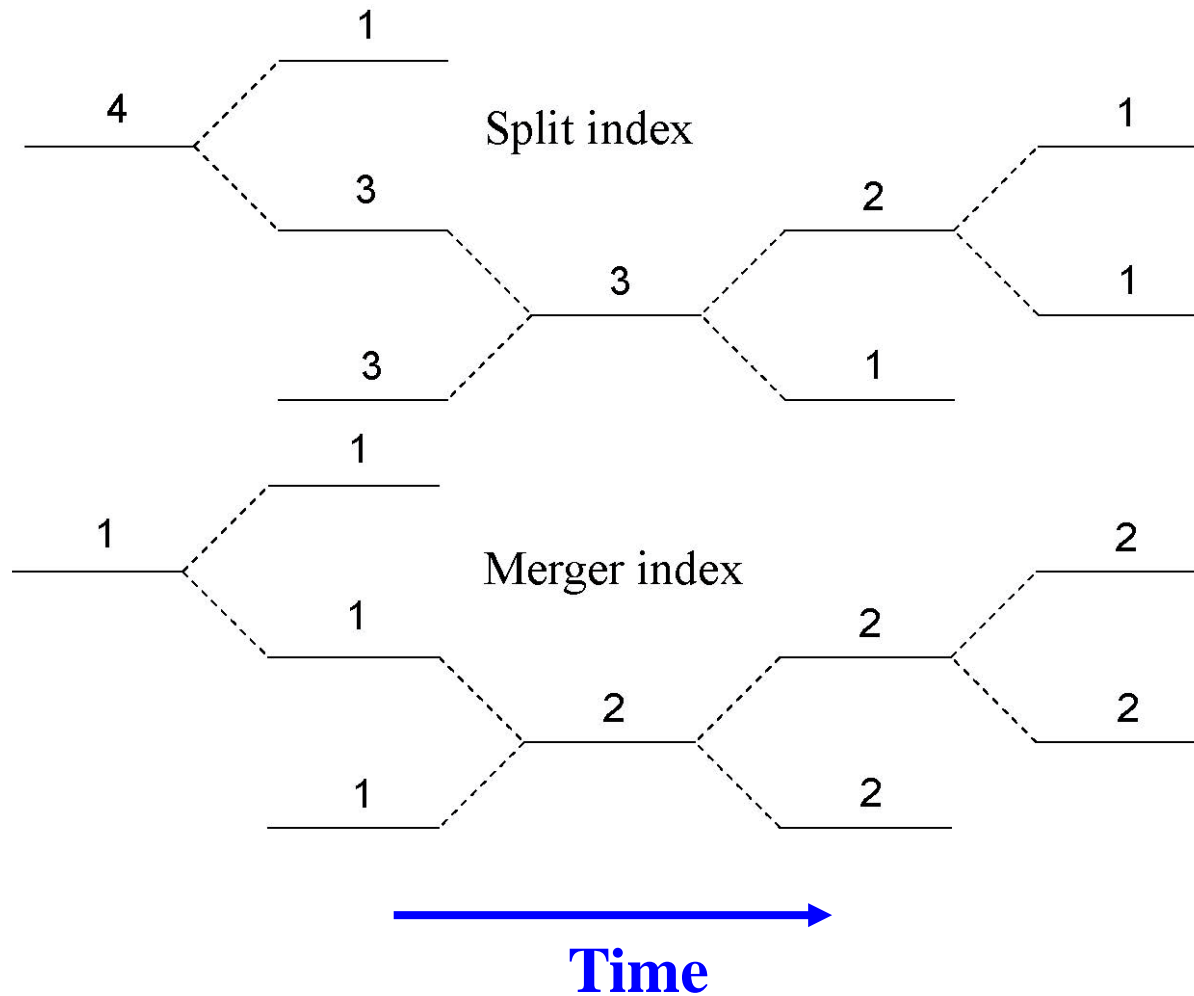
Lozano-Durán & J (2014)

Growth and Decay

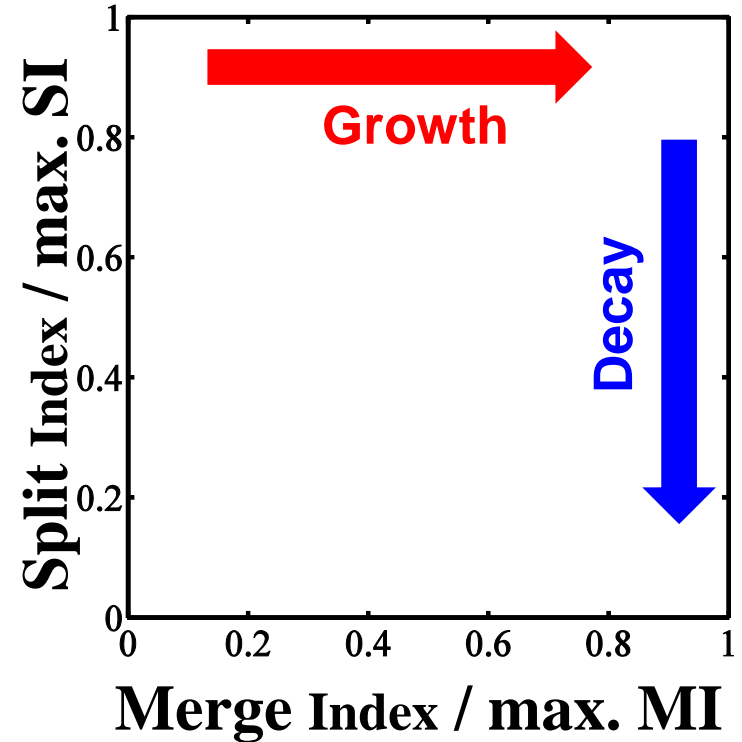
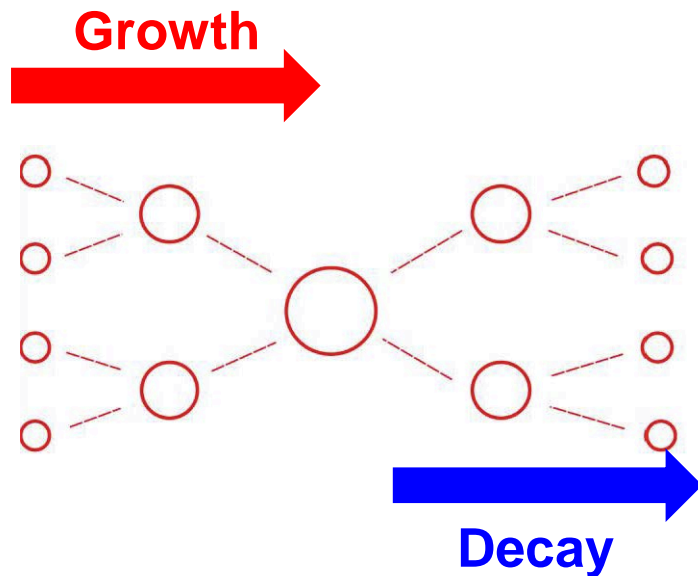


$Re_{\tau} = 950-4200$

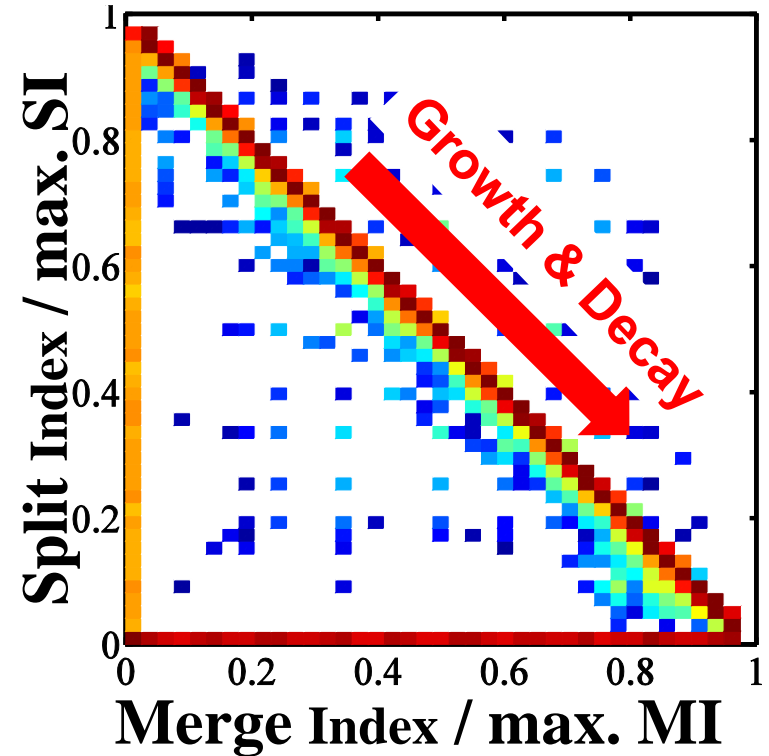
Merger and Split Indices



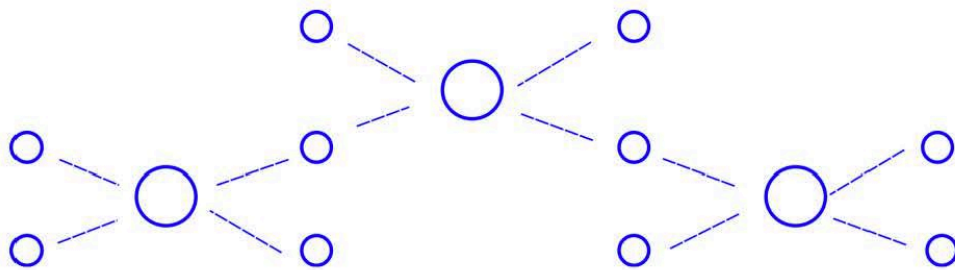
Growth and Decay (the theory)



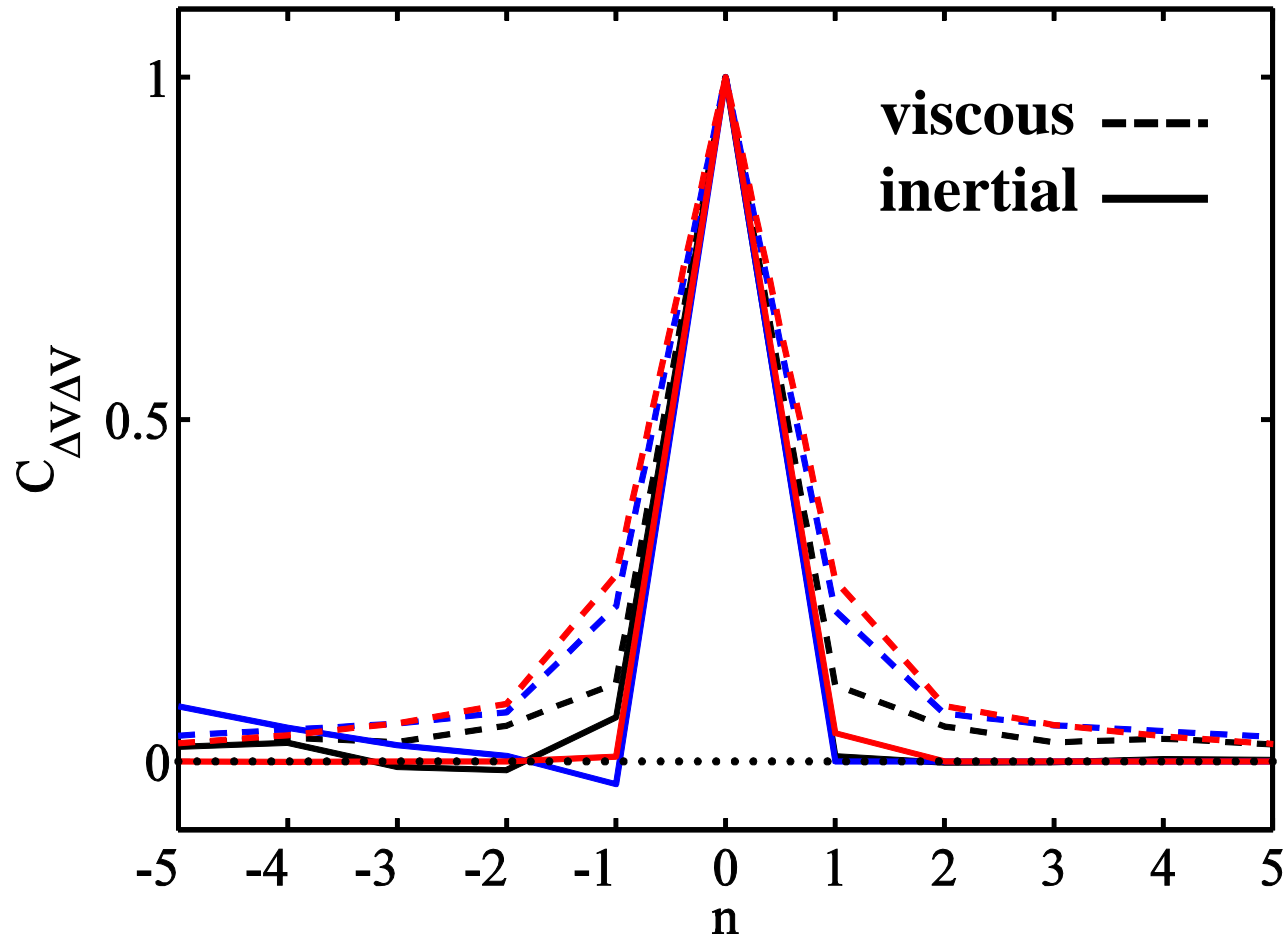
Growth and Decay (the data)



Growth & Decay



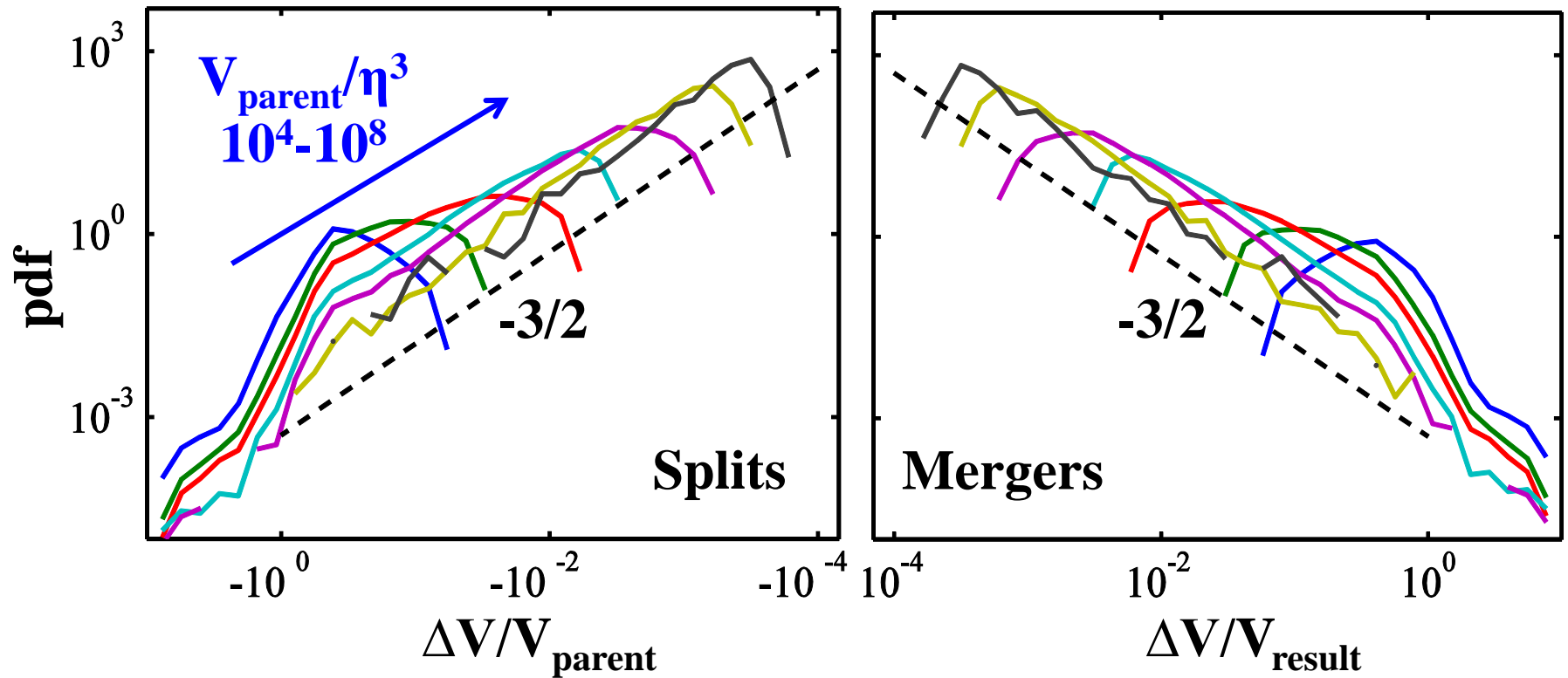
Mergers and Splits are **Markovian**



$Re_\tau = 950-4200$; “detached”

Lozano-Durán & J (2014)

Relative Volume Increments

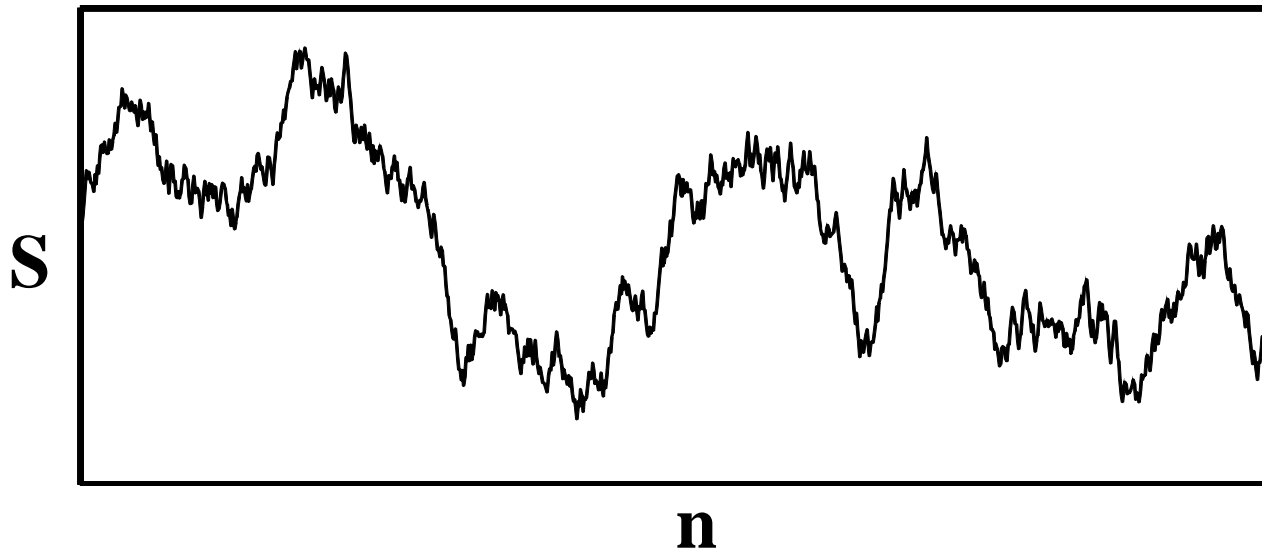


$Re_{\tau} = 4200$; "detached"

Lozano-Durán & J (2014)

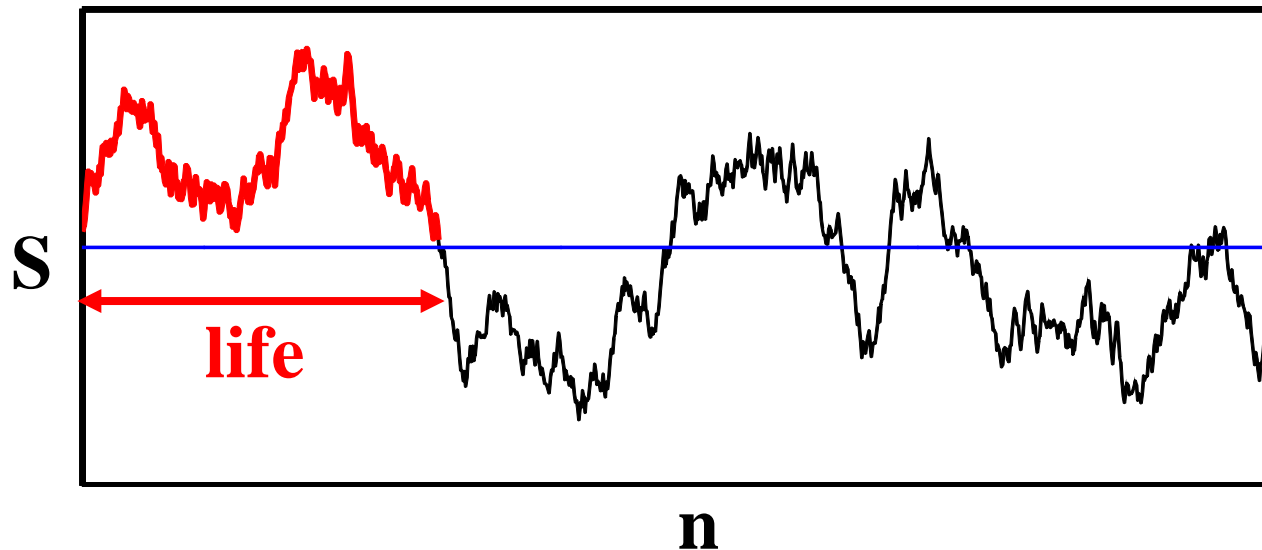
A fair coin toss (martingale)

$$S_n = S_{n-1} \pm 1\$$$

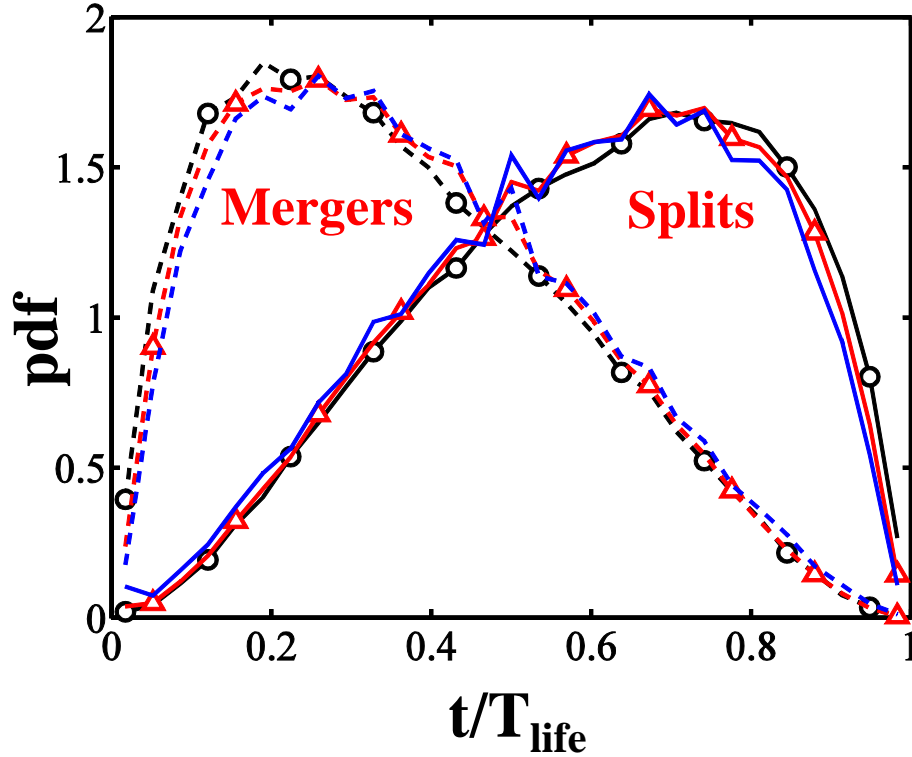


A fair coin toss (martingale with ruin)

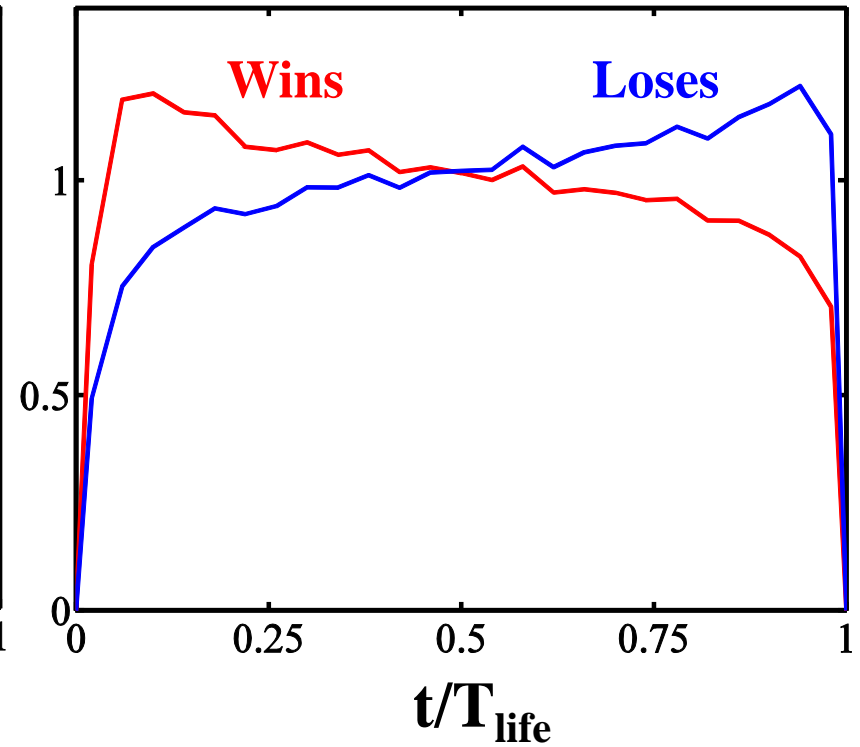
$$S_n = S_{n-1} \pm 1\$$$



Growth and Decay



Channel



Martingale

Martingales with ruin

Additive: $S_{n+1} = S_n + r_n$

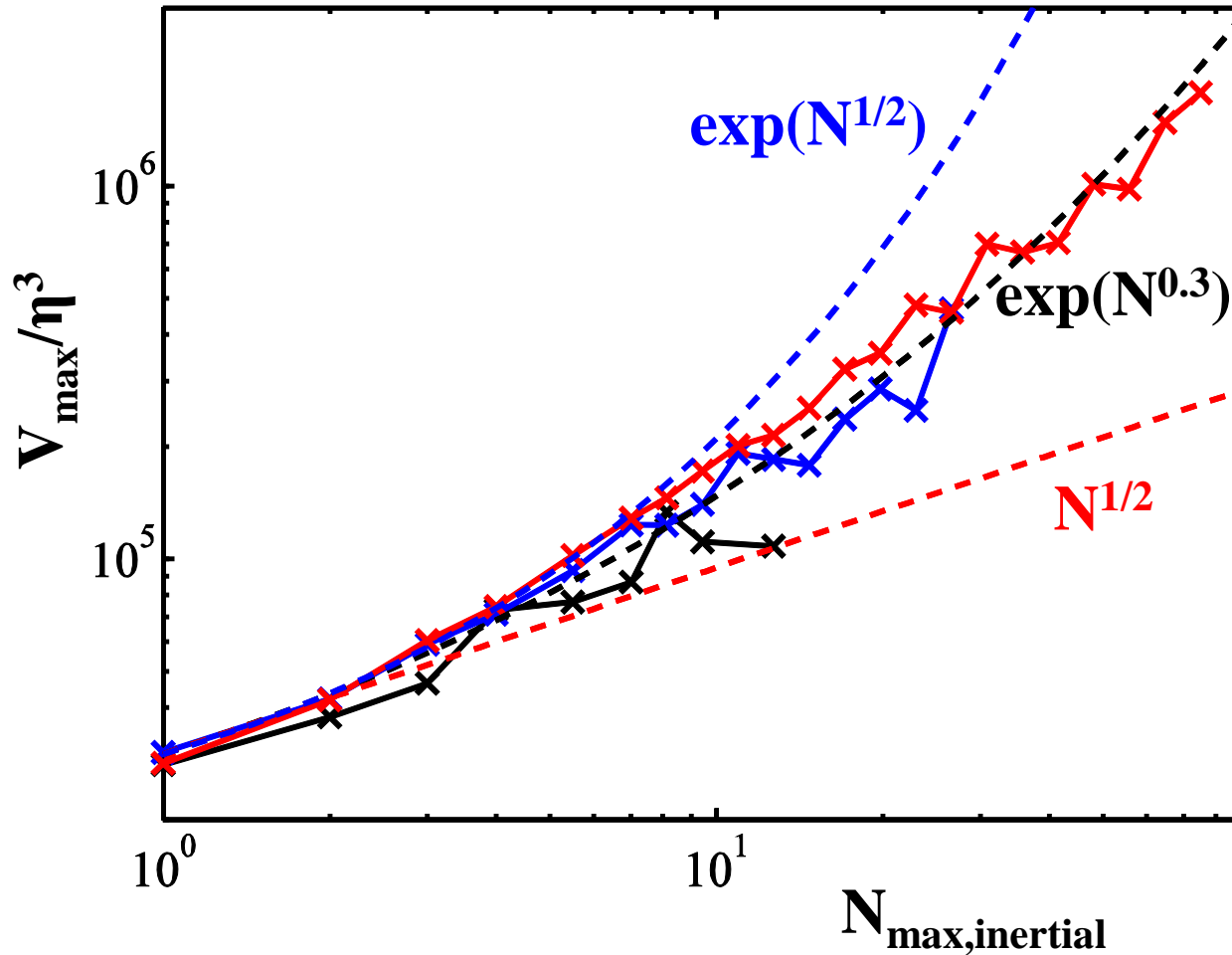
$$S_{\max} \sim N_{\max}^{1/2}$$

Multiplicative: $S_{n+1} = S_n * r_n$

$$\log(S_{n+1}) = \log(S_n) + \log(r_n)$$

$$S_{\max} \sim \exp(N_{\max}^{1/2})$$

Volume versus Life



$Re_\tau = 950-4200$; “inertial”

Lozano-Durán & J (2014)

Summary

- There is a **forward energy** cascade that **takes time**
- It **crosses** the inertial range **incrementally**
- Its velocity is given by the **local eddy turnover**
- It is **intermittent** (or $u_{\max} \approx u'$)
- The **momentum flux** in channels also **cascades**
- It is **not intermittent**
- It can be followed in **individual eddies**
- It “**resembles**” a **martingale with ruin**

Thank you

The Viscous Layer is **Small**

Channel (U): $Re_\tau = 2000$, $y^+=15$

