Turbulence Still Surprises: Explorations Using a 1D Model

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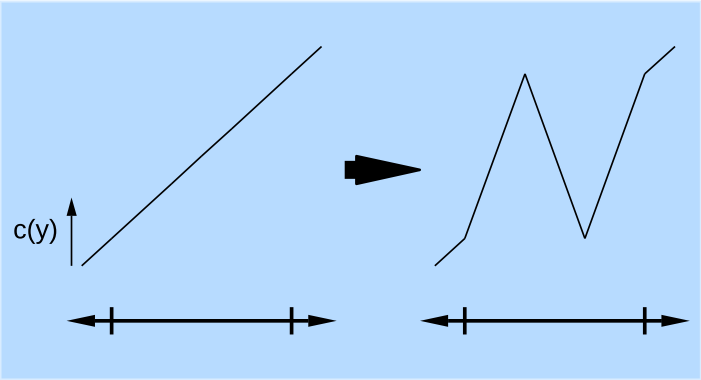
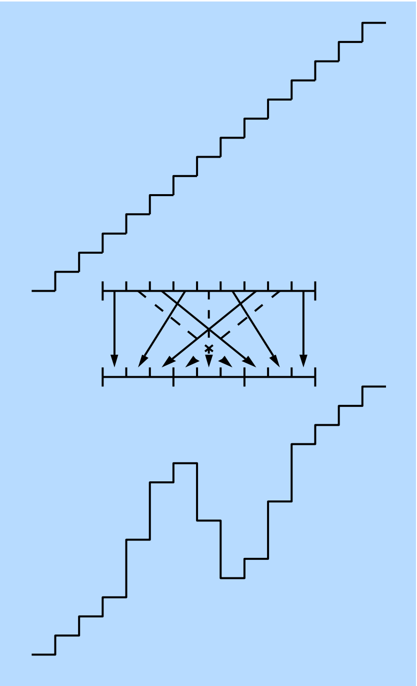
Outline of presentation

- **Overview of the modeling approach**
- Unexpected large-scale effects in pipe mixing
- Re dependence of differential molecular diffusion
- Sensitivity of moderate Re convective entrainment to molecular transport
- Spontaneous layer formation in buoyant stratified flow
- Slow deposition of high-inertia particles
- Counterintuitive dependence of jet spreading on molecular diffusivity
- Conclusions

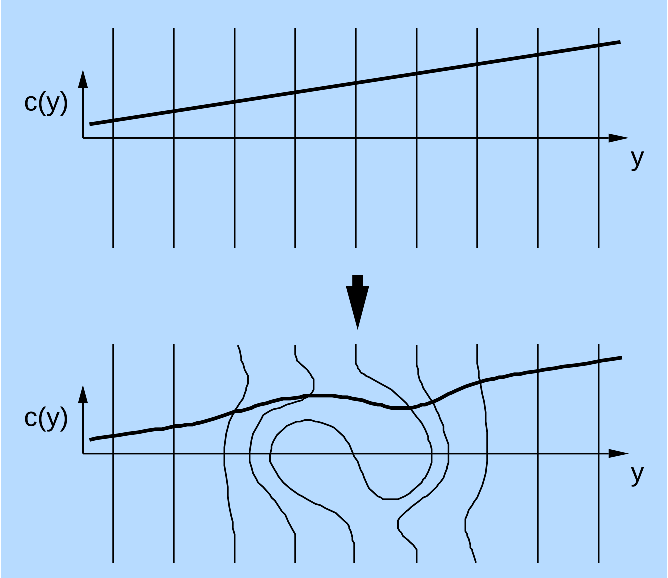
Advection is modeled as a sequence of *triplet maps* that preserve desired advection properties, even in 1D



The triplet map captures compressive strain and rotational folding effects, and causes no property discontinuities



The triplet map is implemented numerically as a permutation of fluid cells (or on an adaptive mesh)



This procedure imitates the effect of a 3D eddy on property profiles along a line of sight

The triplet map (1D eddy)

- <u>moves</u> fluid parcels <u>without intermixing</u> their contents
- <u>conserves</u> energy, momentum, mass, species, etc.
- reduces fluid separations by at most a factor of 3
    - <u>Conjecture</u>: It is optimal in this respect

There are different ways to specify the map sequence during a simulation

- Linear-Eddy Model (LEM): Map occurrences and properties (size, location) are sampled from fixed distributions
  - Parameters determining these assignments based on the turbulent flow state at each location must be provided as input
  - LEM evolves scalar profiles but not velocity, hence is a turbulent mixing model, not a turbulence model
- One-Dimensional Turbulence (ODT): Eddy sampling is based on the flow state evolved by the model
  - After parameter adjustment, ODT predicts turbulence evolution
  - The required input is the flow configuration (ICs, BCs)
- In either model, the eddies (instantaneous maps) punctuate continuous-in-time advancement of molecular-diffusive transport, chemistry, etc. For example:

$$u_t = \nu u_{yy} + \text{'eddies'} \qquad \theta_t = \kappa\theta_{yy} + \text{'eddies'}$$

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Simple configuration: one eddy size, sinusoidal initial scalar – what happens?

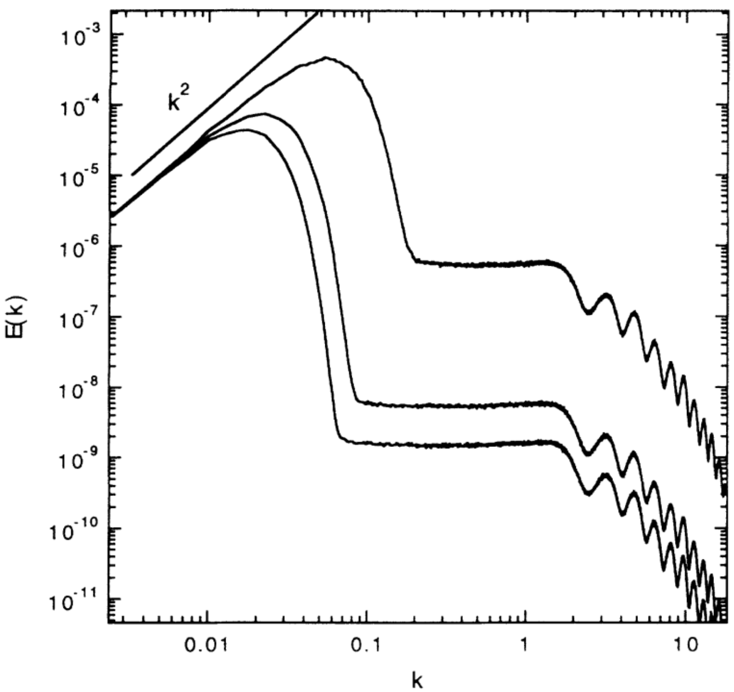
Evolve $\theta_t = \kappa\theta_{yy}$ + ‘eddies’ with

- $\theta(y,0) = \sin(2\pi y/L)$
- Randomly placed triplet maps, all size L
- High map frequency (eddy transport >> κ)
- Domain size >> L, periodic boundary conditions

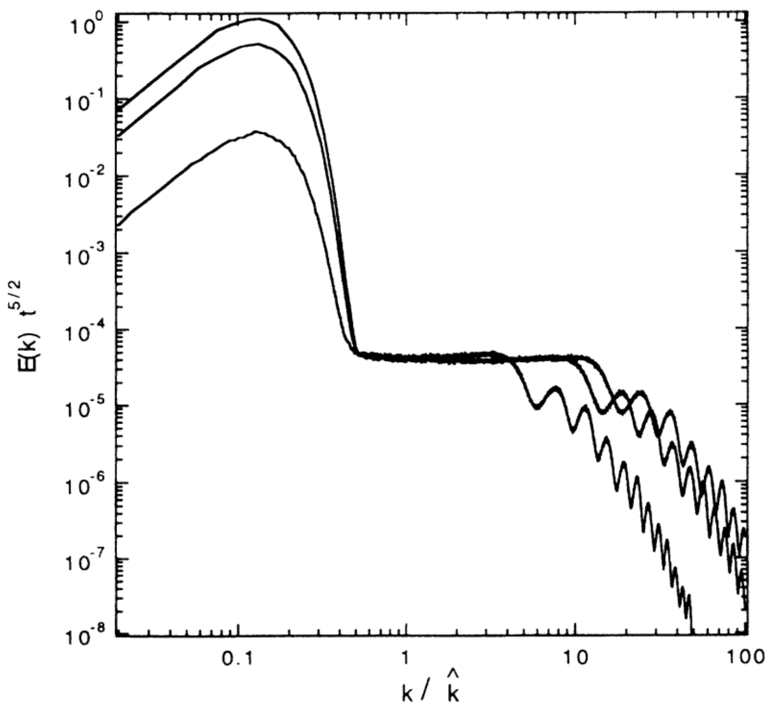
What is the time evolution of:

- Scalar variance?
- Scalar power spectra?

The result was surprising (amazing!) – then an explanation was found



top to bottom: increasing t



analysis predicts the collapse seen in this scaled plot

Pipe flow measurements motivated by these results illustrate the cause of this behavior

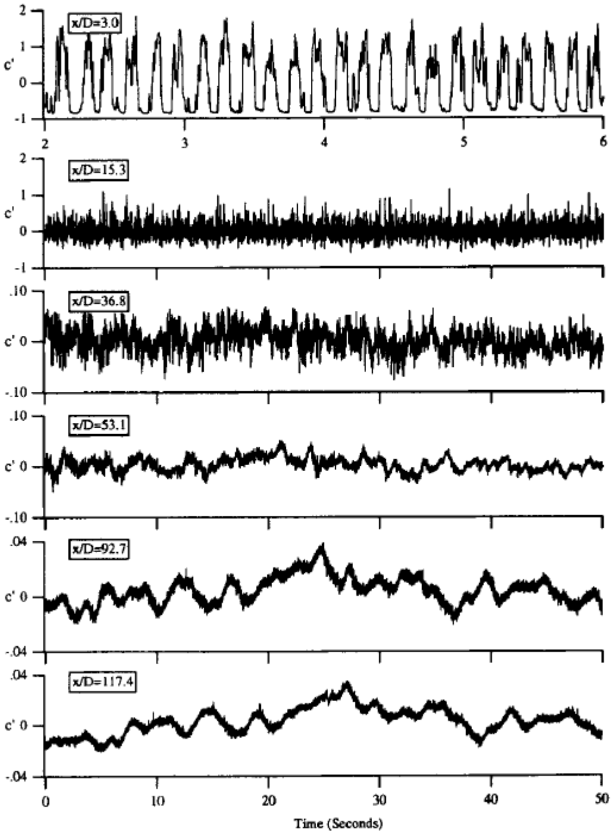
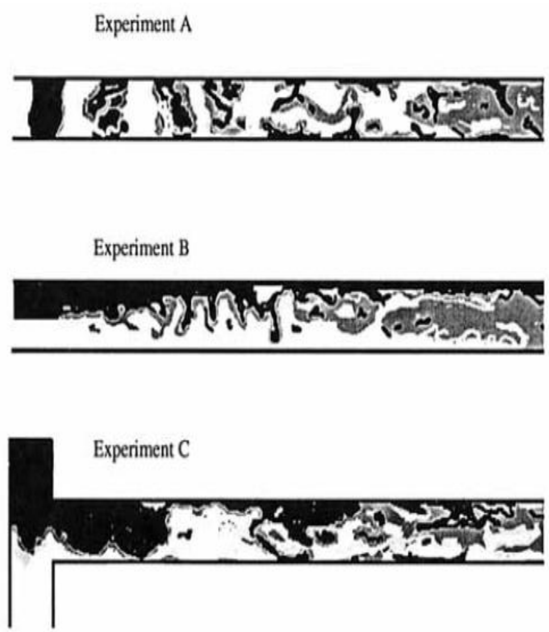


Figure 3. Time series of measured scalar field at the center of the pipe for experiment A.

A 4-s period is shown for $x/D = 3.0$ to show the idealized inlet condition achieved. At all other locations a 50-s time series is shown.



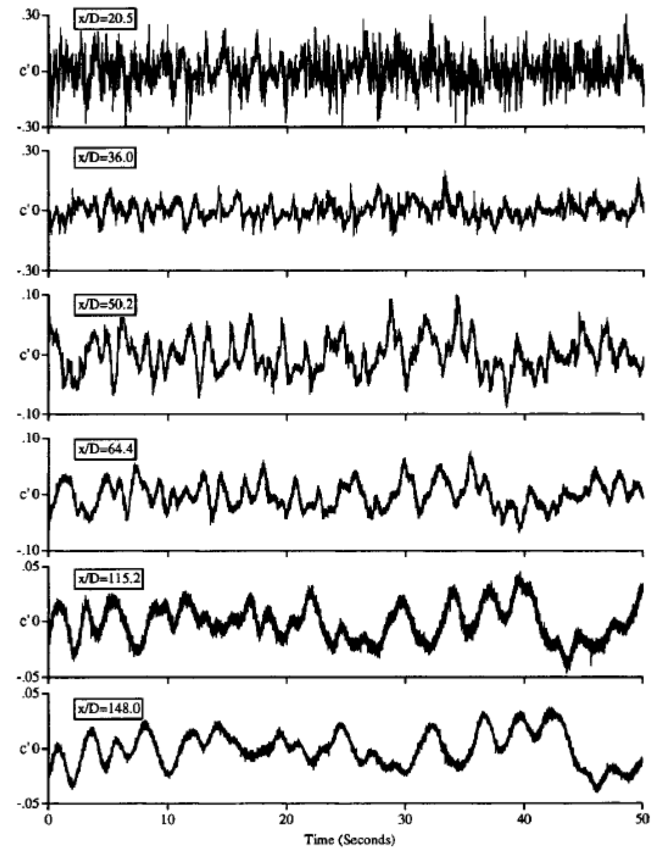
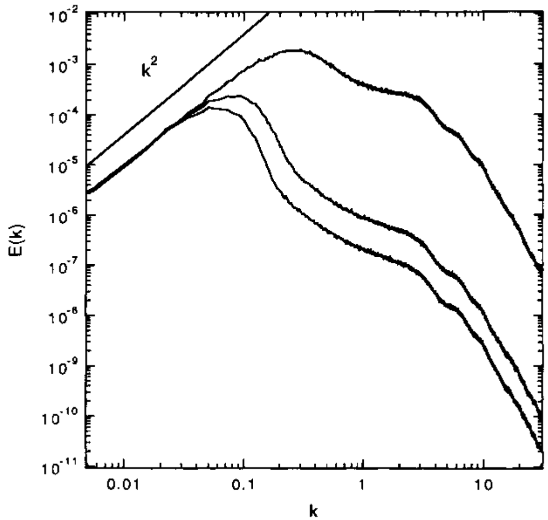
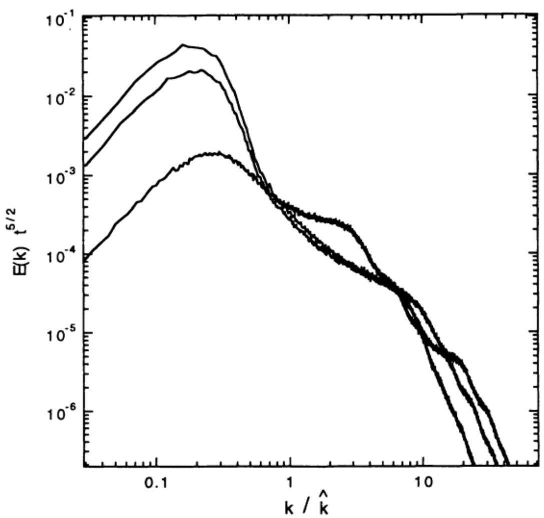


Figure 11. Time series of measured scalar field at the center of the pipe for experiment C.

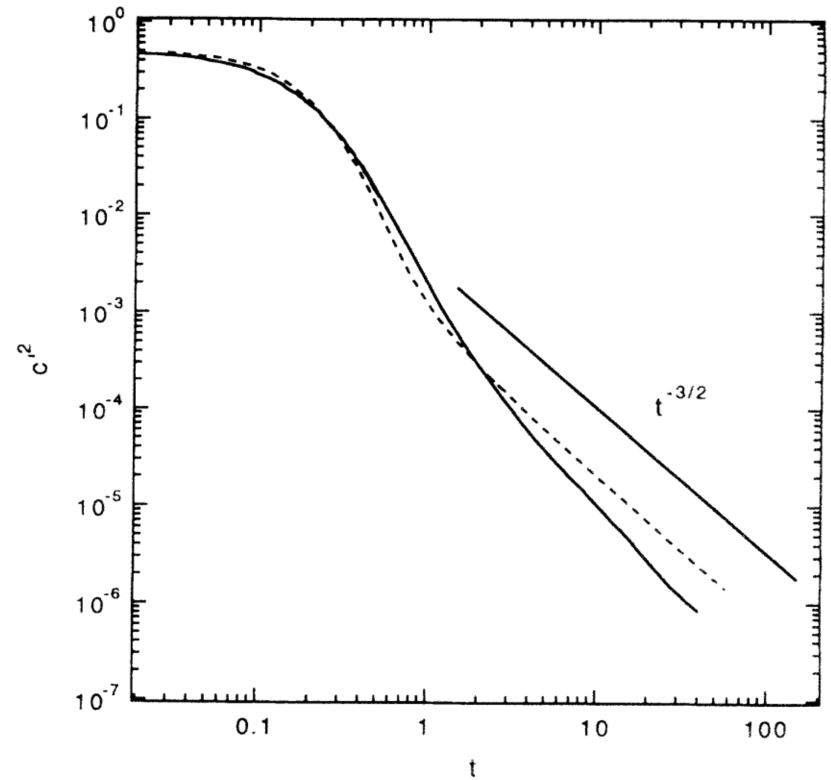
Guilkey, McMurtry, and Klewicki, 1997

Simulations were performed for a ‘pipe-like’ map-size distribution





Analysis predicts $t^{-3/2}$ scalar-variance decay



— — one map size

—— pipe-like size distribution

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Scalar power-spectrum measurements exhibit the predicted features

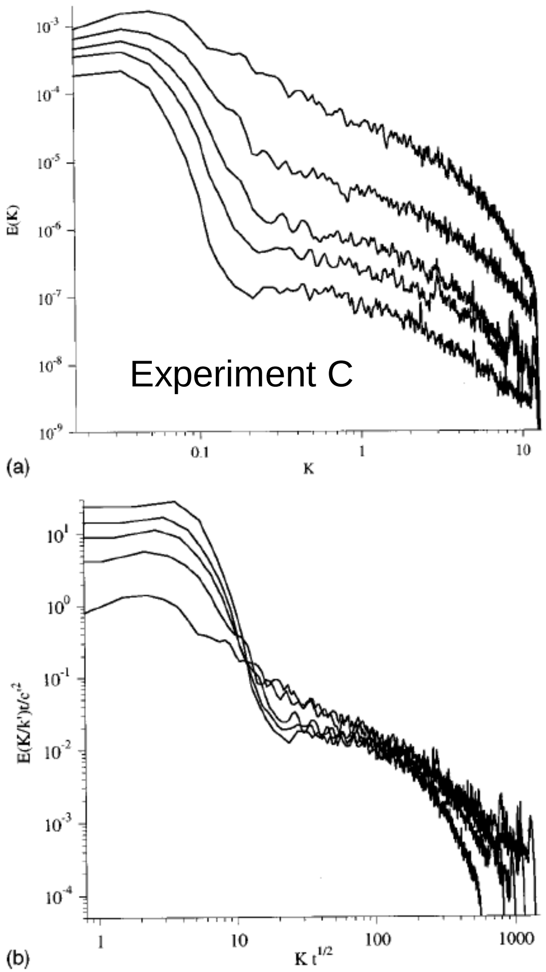


FIG. 5. (a) Power spectral densities of scalar fluctuations, experiment 2. Axial locations (from top to bottom) are $x/D = 20.5$, 36.0, 50.2, 64.4, and 90.3. (b) Spectra subject to ''equilibrium'' range scalings, indicating self-preserving behavior.

Pipe measurements show a transition from exponential to power-law variance decay

Brodkey, 1966, 'confirmed' exponential decay (Corrsin's batch-reactor analysis) to x/D = 30

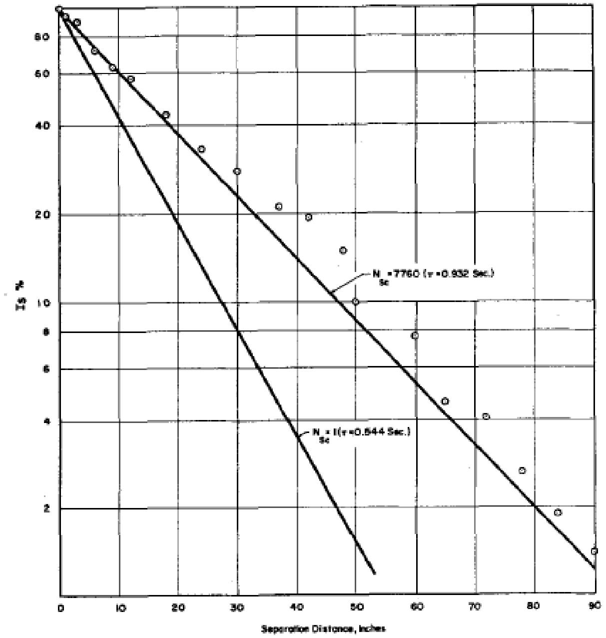
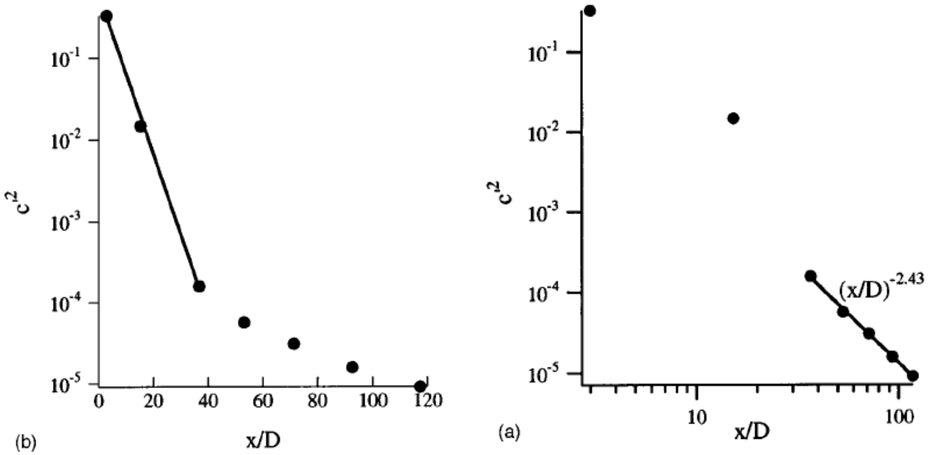
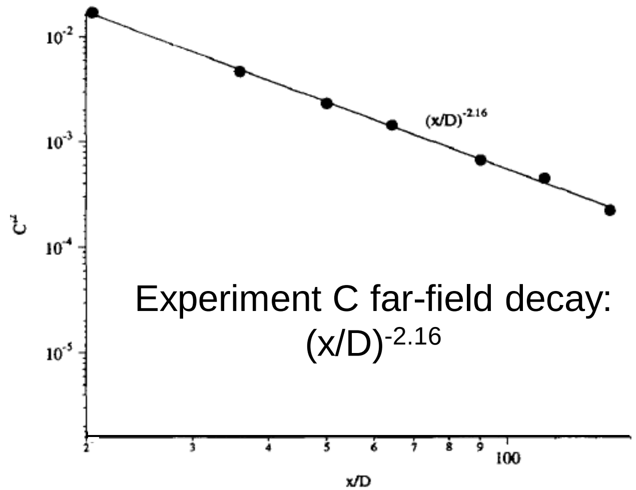


Fig. 1. Intensity of segregation.

Experiment A: near-field exponential, far-field $(x/D)^{-2.43}$





Experiment C far-field decay: $(x/D)^{-2.16}$

Near-field decay depends on initialization – <u>the only robust result is the far-field power law</u> (with a non-universal exponent)

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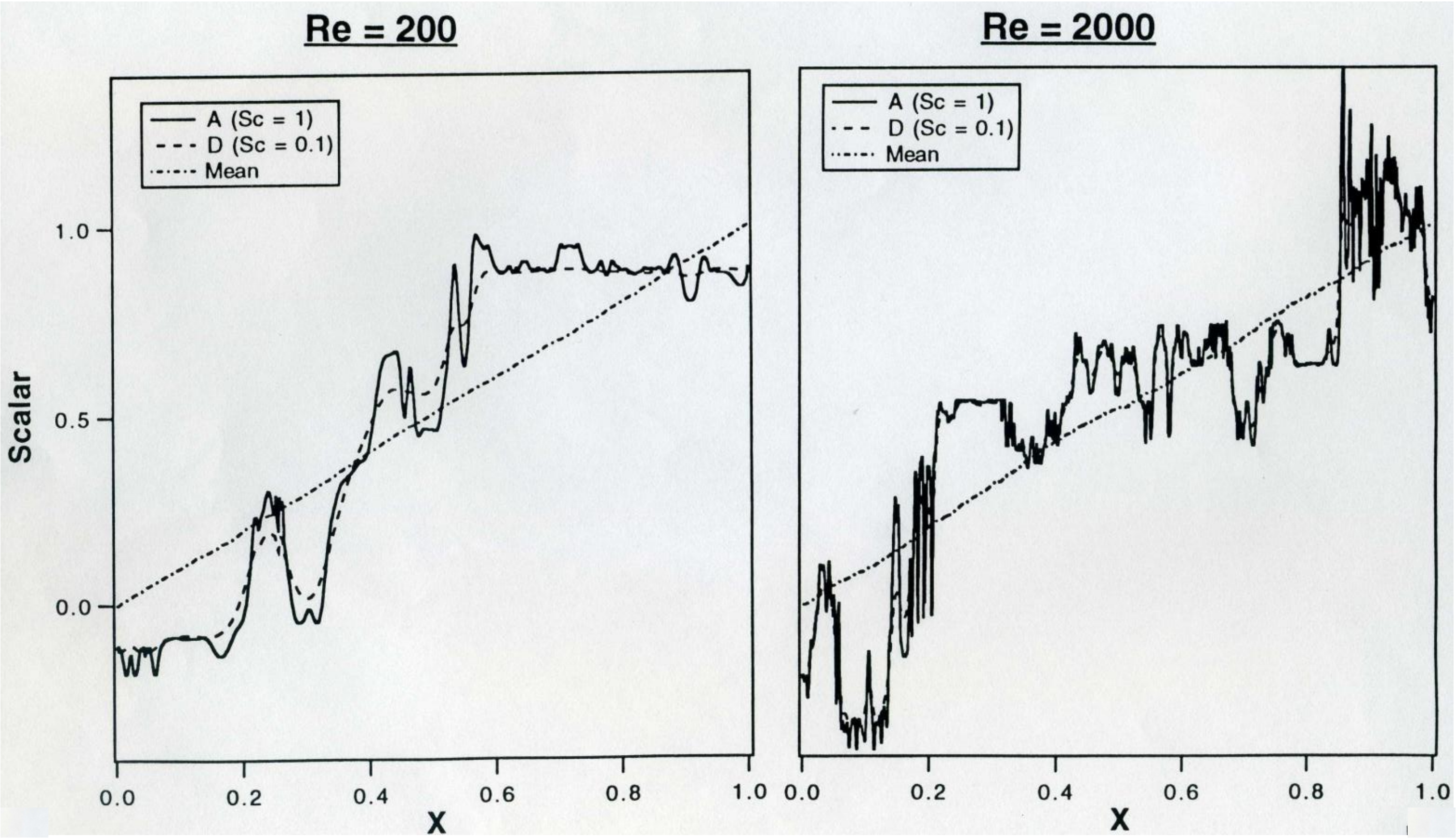
Linear-eddy model (LEM): distribution of eddy sizes, obeys inertial-range scalings

Evolve $\theta_t = \kappa \theta_{yy}$ + 'eddies' where

- The map distribution is spatially uniform (homogeneous turbulence)
- Map sizes range from η (Kolmogorov microscale) to L
- Map size PDF $f(\ell)$ is determined by $\kappa_e(\ell) \sim \ell\, v(\ell) \sim \ell^{4/3}$
- Need an input value of $\kappa_e(L)$ to set the overall map frequency
- Non-dimensional parameters: Re ~ $(L/\eta)^{4/3}$, Pe ~ $\kappa_e(L) / \kappa$
- Sc ~ Pe / Re, which implies a kinematic viscosity (though no velocity!)

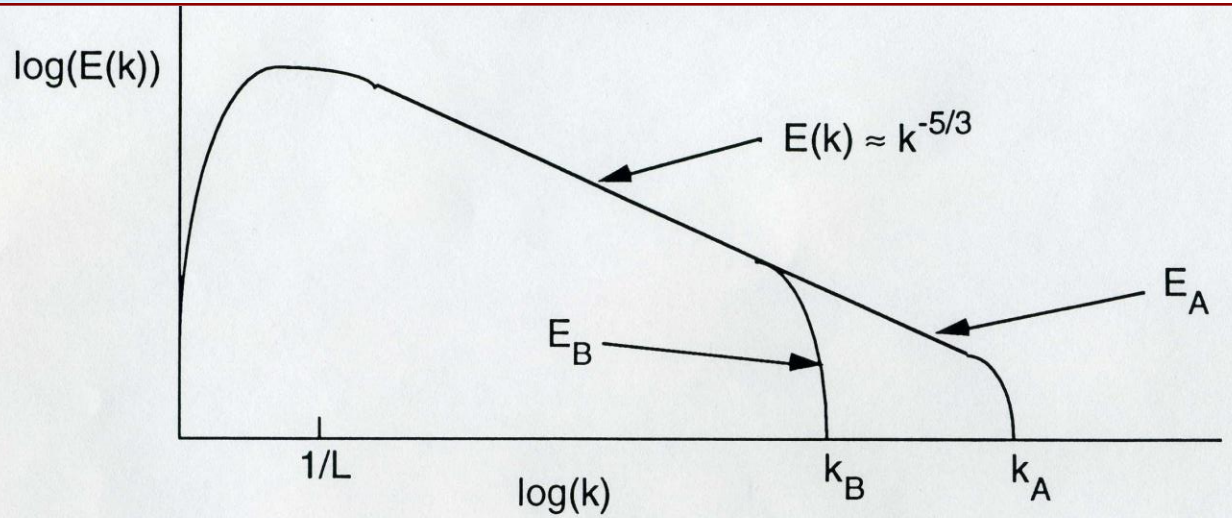
Goal: Analyze the time evolution of two scalars with identical initial spatial distributions but different diffusivities

Do differential molecular diffusion effects vanish with increasing Re as quickly as supposed?



Bilger and Dibble, 1982, proposed z' ~ Re^{-1}, where z = c_A - c_D

A spectral picture suggests slower falloff of z'



- Obukhov-Corrsin scale: $L_C \approx (Pe)^{-3/4}$
- $c'^2 = \int E(k)\, dk$
- Since A and B only separate in the wavenumber range $k_B < k < k_A$ we get:

$$z'^2 \sim \int_{k_B}^{k_A} k^{-5/3} dk \sim k_A^{-2/3} - k_B^{-2/3} \sim Pe_A^{-1/2} - Pe_B^{-1/2} \sim Re^{-1/2}(Sc_A^{-1/2} - Sc_B^{-1/2})$$

$$\Rightarrow z' \sim Re^{-1/4}$$

This was confirmed using LEM (Cremer, Kerstein, and McMurtry, 1995) and then using DNS (Nilsen and Kosaly, 1998)

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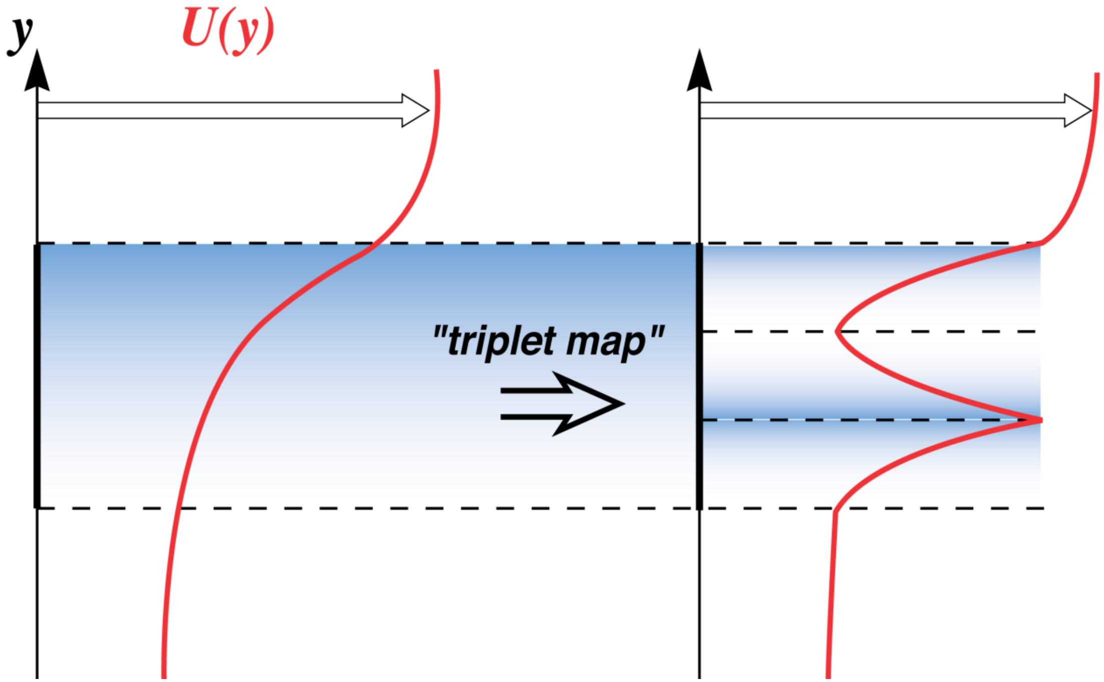
In ODT, the triplet map amplifies shear, inducing an *eddy cascade* (feedback mechanism)

- **The key to model performance is the eddy selection procedure**
- Eddy likelihood, in a random sampling procedure, is governed by local shear
- When an eddy occurs, the local shear is amplified, which modifies eddy likelihoods



High shear at small scales drives small eddies, leading to an eddy cascade

(In LEM, inertial-range-cascade scaling is hard-wired)

ODT eddy selection is based on the mixing-length concept, applied locally

- Each possible eddy, defined by eddy spatial location and size (S), is assigned a time scale τ based on local energetics (shear, buoyancy, etc.)

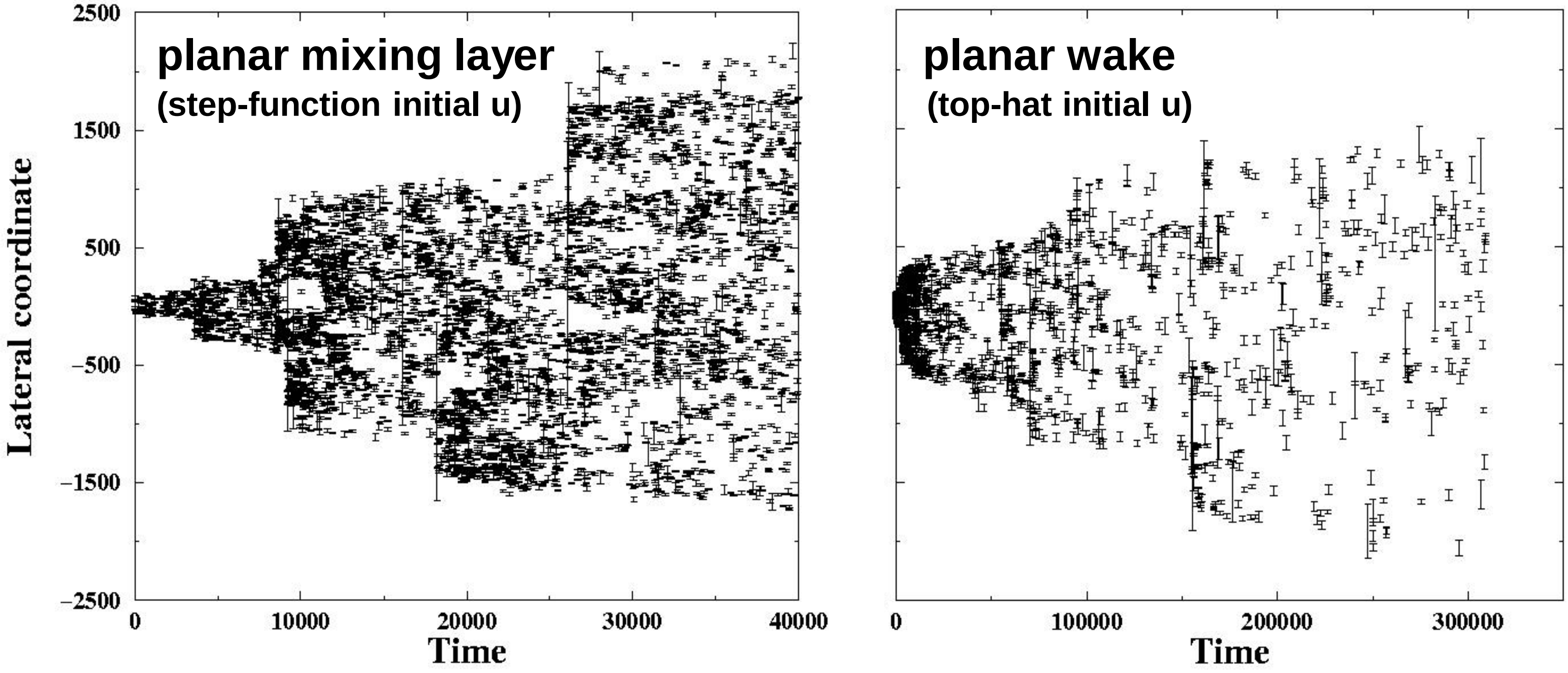
- This defines an eddy velocity S/τ and energy density $\rho\,(S/\tau)^2$

- The set of τ values determines an eddy rate distribution from which eddies are sampled

- Unlike conventional mixing-length theory, this procedure is local in space and time (no averaging) and is applied to all eddy sizes S (multi-scale) rather than a single selected S value ('mixing length')

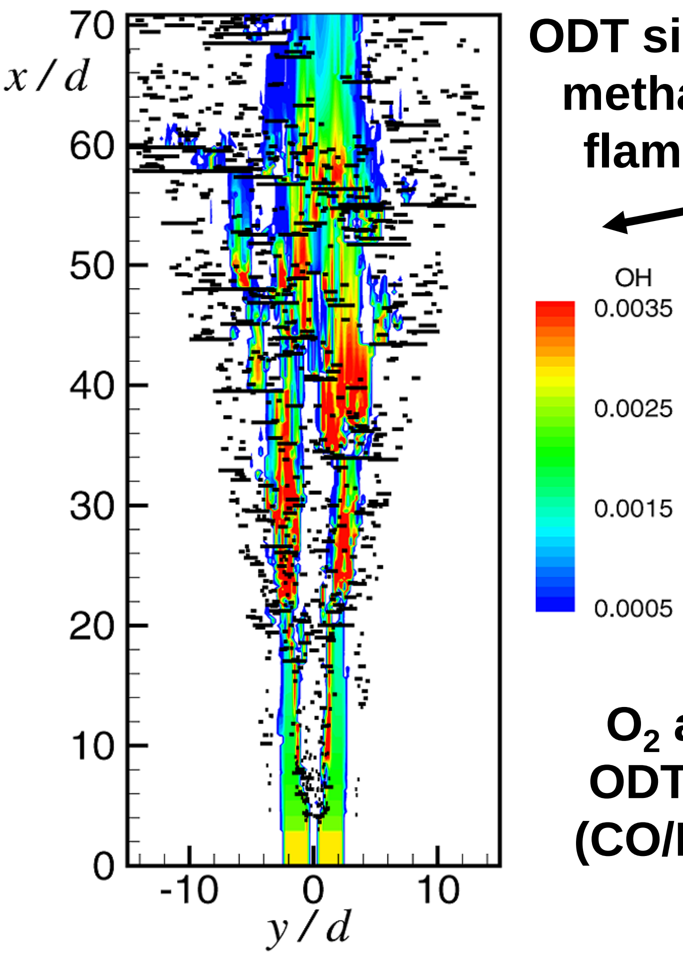
ODT simulations provide detailed flow-specific representations of turbulence

These simulations are based on time advancement of $u_t = \nu u_{yy}$ with flow-specific initial u profiles (see below), plus eddies



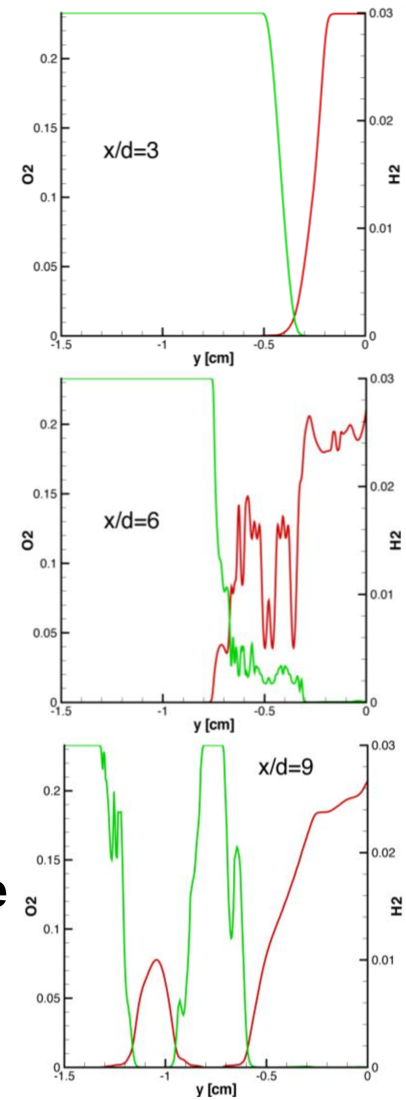
- **Each vertical line shows the spatial extent of an eddy**
- **Horizontal location is its time of occurrence**
- **Units are arbitrary**

LEM and ODT resolve advective-diffusive-reactive couplings and hence all flame regimes



ODT simulation of a piloted methane-air jet diffusion flame (Sandia flame D)

O_2 and H_2 profiles from an ODT simulation of a syngas ($CO/H_2/N_2$) jet diffusion flame



In a water-tank experiment, bottom heating was a surrogate for cloud-top cooling

Experimental setup (B. Sayler & R. Breidenthal 1998):

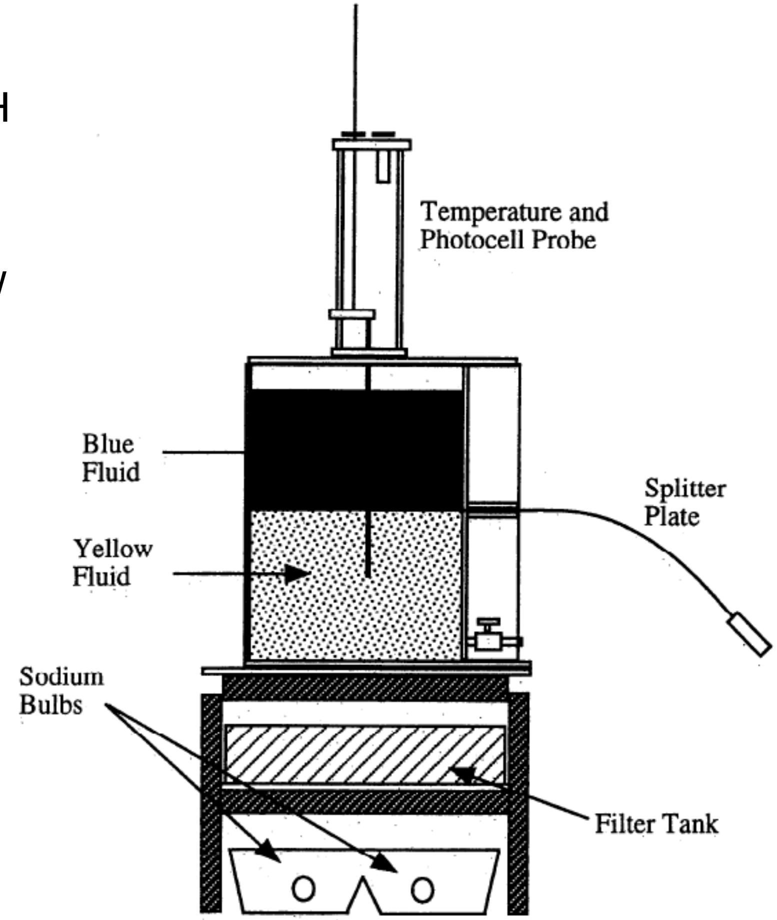
- water with pH indicator, blue (yellow) at high (low) pH
- upper layer is blue and warm (stable layer interface)

Procedure: radiant heating of the upper layer from below

- drives turbulent convection in the upper layer
- lower fluid is entrained, deepening the upper layer

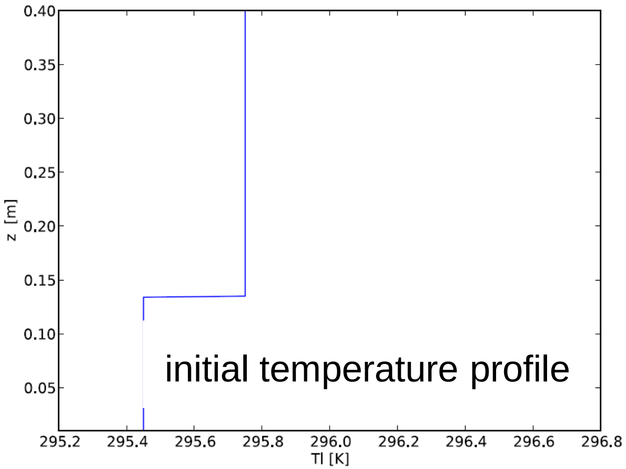
Alternate setup:

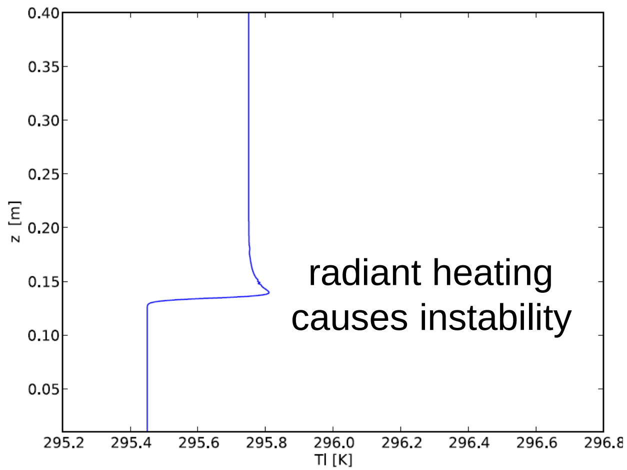
- for initial stratification, dissolve sugar in lower layer
- this tests sensitivity to molecular diffusion

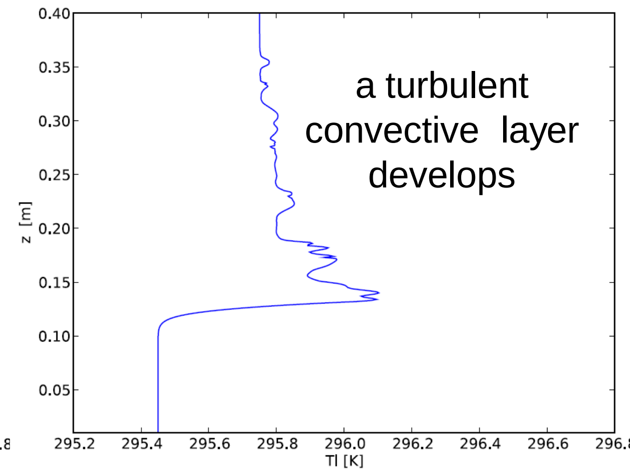


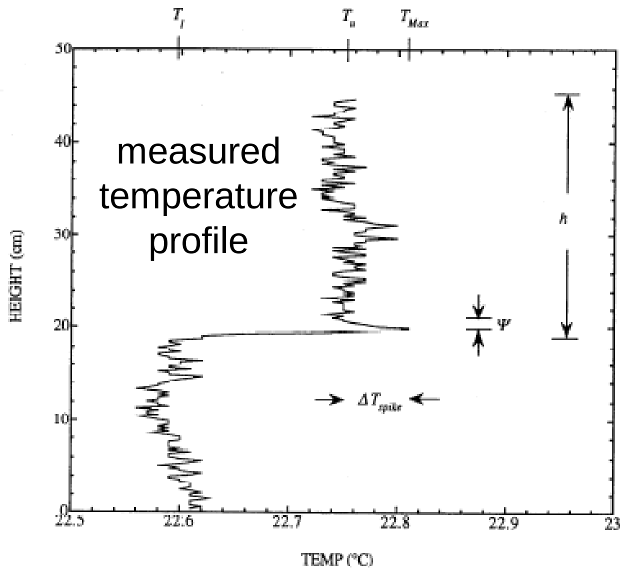
Key feature of this configuration: Convective forcing is adjacent to the entrainment zone

ODT captures salient features of flow evolution in the experimental configuration

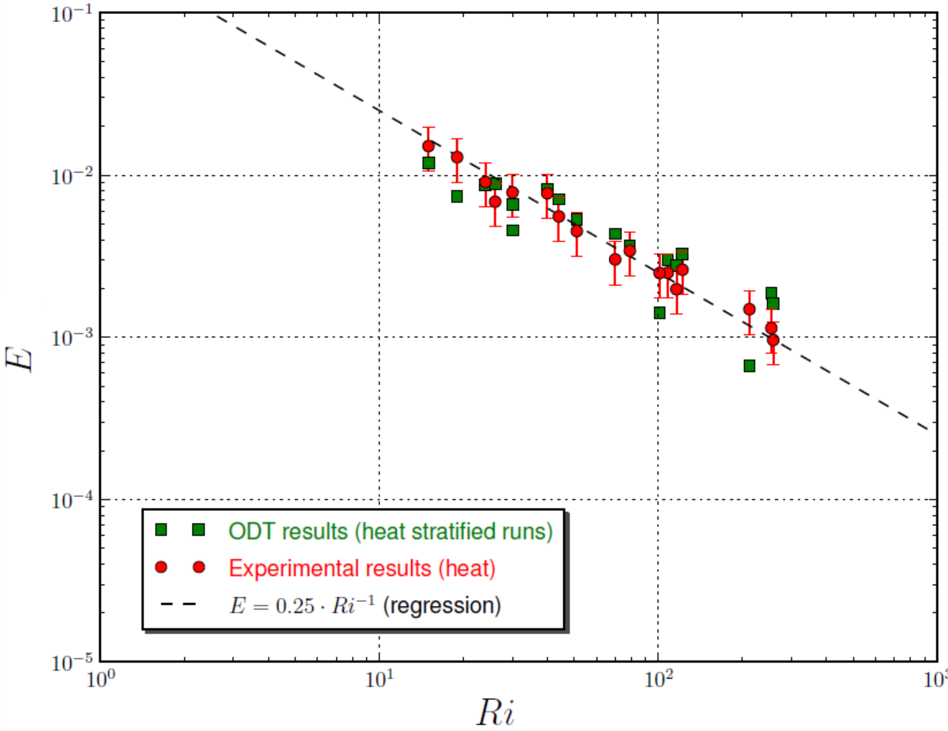


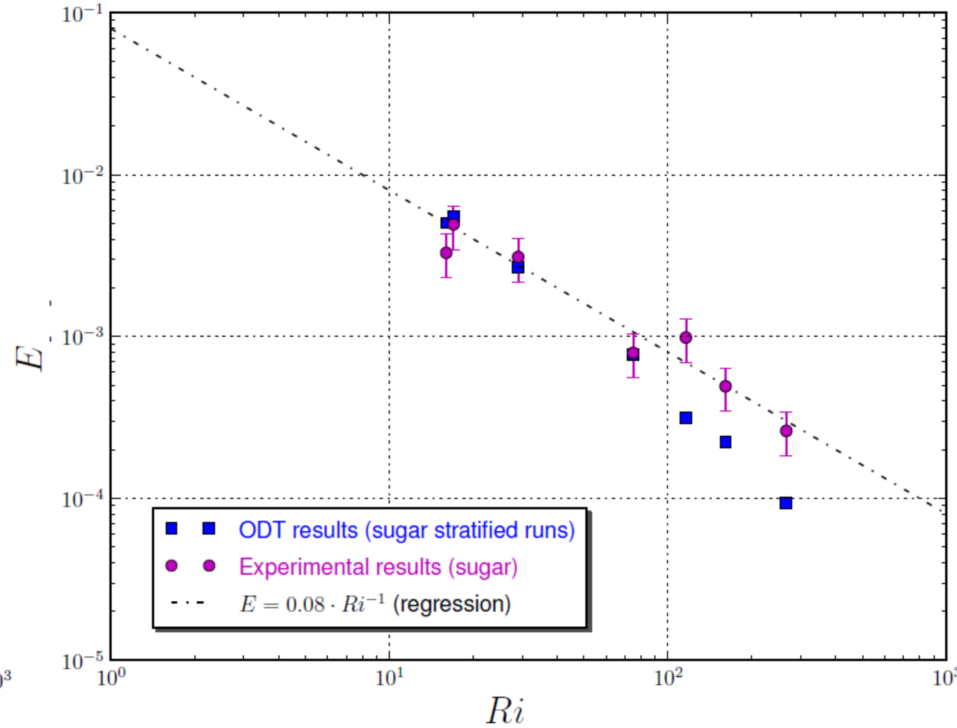






The measured dependences of entrainment on Ri and molecular diffusivity are reproduced





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Scaling analysis implies that the dependence on molecular diffusivity vanishes at cloud scales

Condition for vanishing dependence of the entrainment rate on molecular diffusivity:

$$\frac{s^4 q \alpha g}{\rho c_p} >> K^3$$

where:

s = radiative absorption depth
q = incident radiative heat flux
α = thermal expansion coefficient
g = gravitational acceleration
ρ = reference density
c_p = heat capacity at constant pressure
K = molecular diffusivity

Clouds have much larger *s* than the convective layer in the experiment, so this condition should be satisfied

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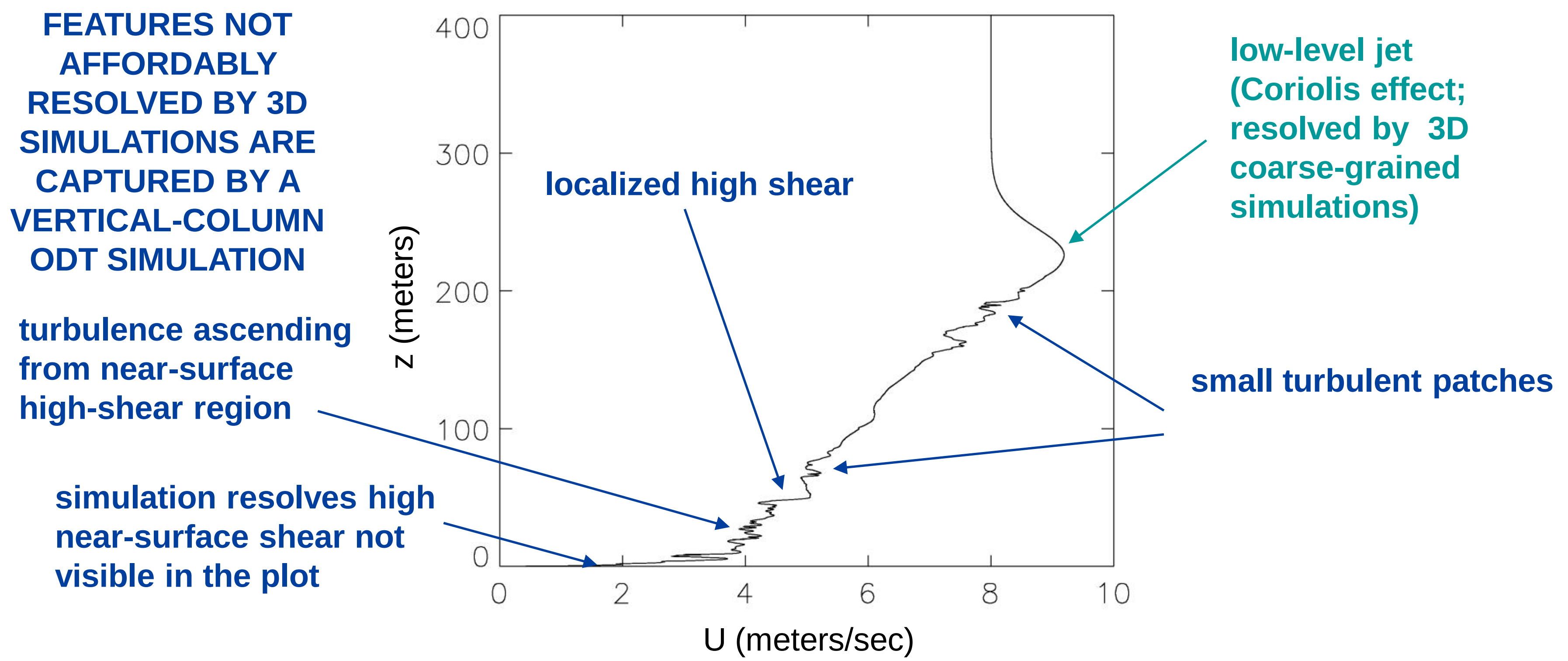
ODT is applicable on km-scale domains – direct application to cloud-top entrainment is underway



- An instantaneous vertical (z) profile of horizontal velocity (U) is shown
- Case shown: Stable cloud-free conditions
- 16000 computational cells, resolving 2.5 cm – requires sub-grid closure
- In progress: Simulations of the cloud-topped boundary layer for interpretation of high-resolution aircraft measurements

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A slow-diffusing stable species can cause layering of a convection process: *double-diffusive instability*

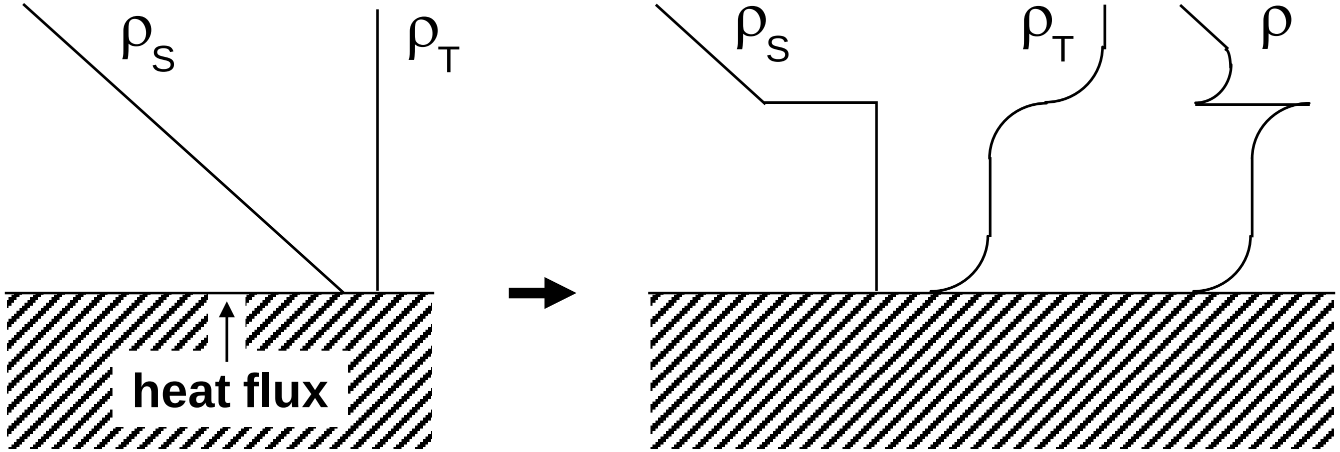
ρ_T **is the density variation due to temperature variation**

ρ_S **is the density variation due to salinity variation**

<u>Initial state</u>: constant temperature, salinity decreases with increasing height (stable, no motion)

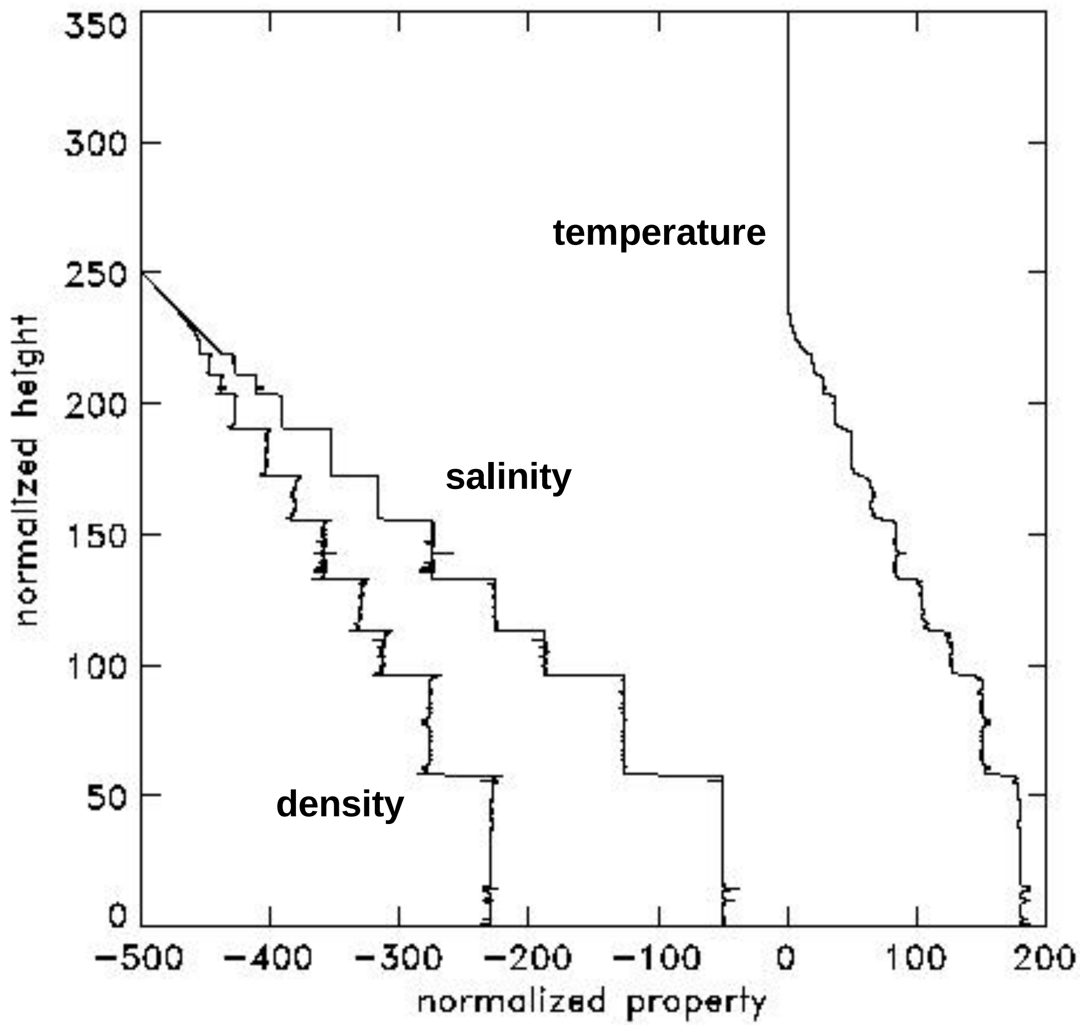
<u>Forcing</u>: heat from below causes gravitational instability leading to turbulent mixing

<u>Role of molecular transport</u>: salt diffusivity is negligible, so stable jump forms, but heat diffuses across, initiating a new turbulent layer above the jump



thermohaline staircase

ODT captures the wide range of dynamically relevant time and length scales



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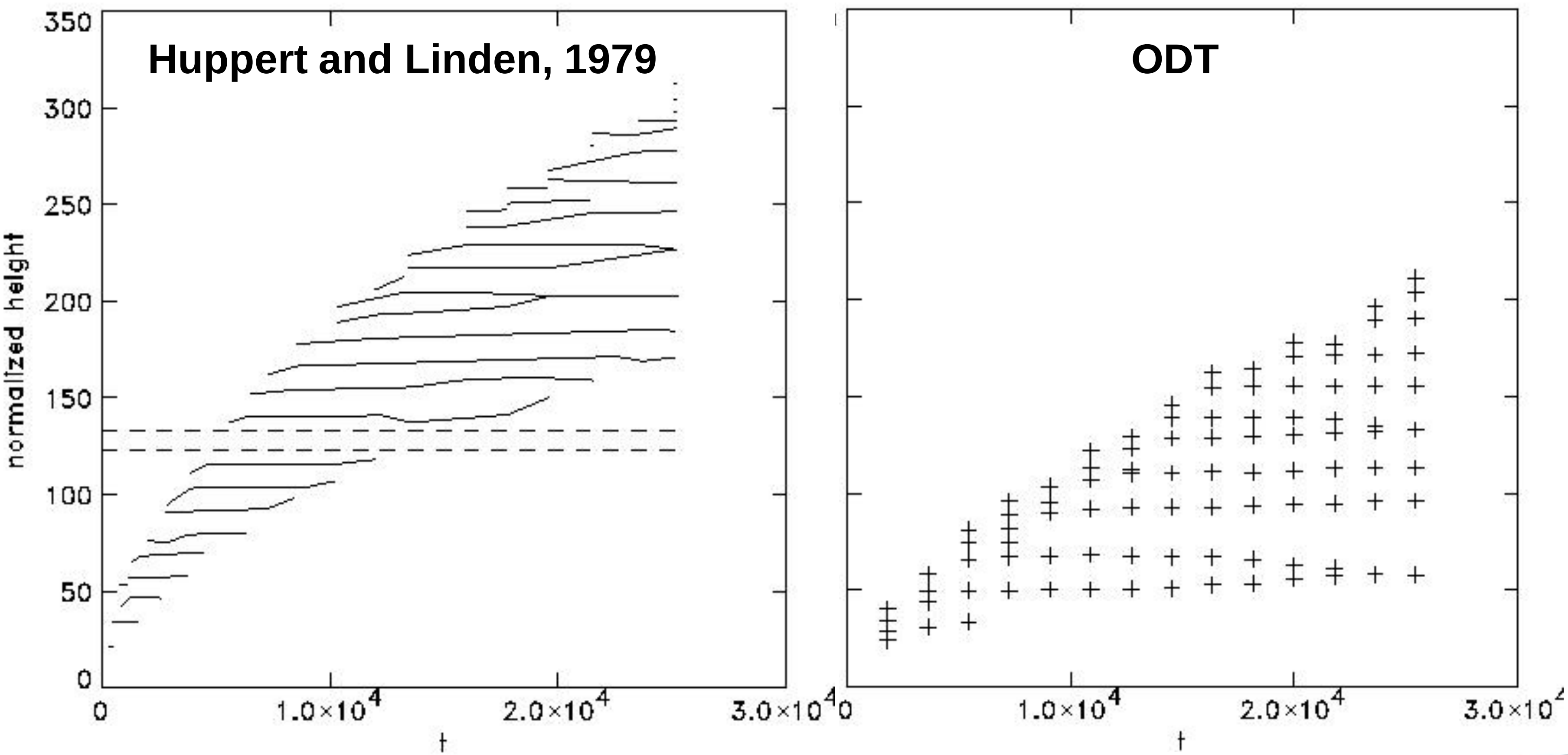
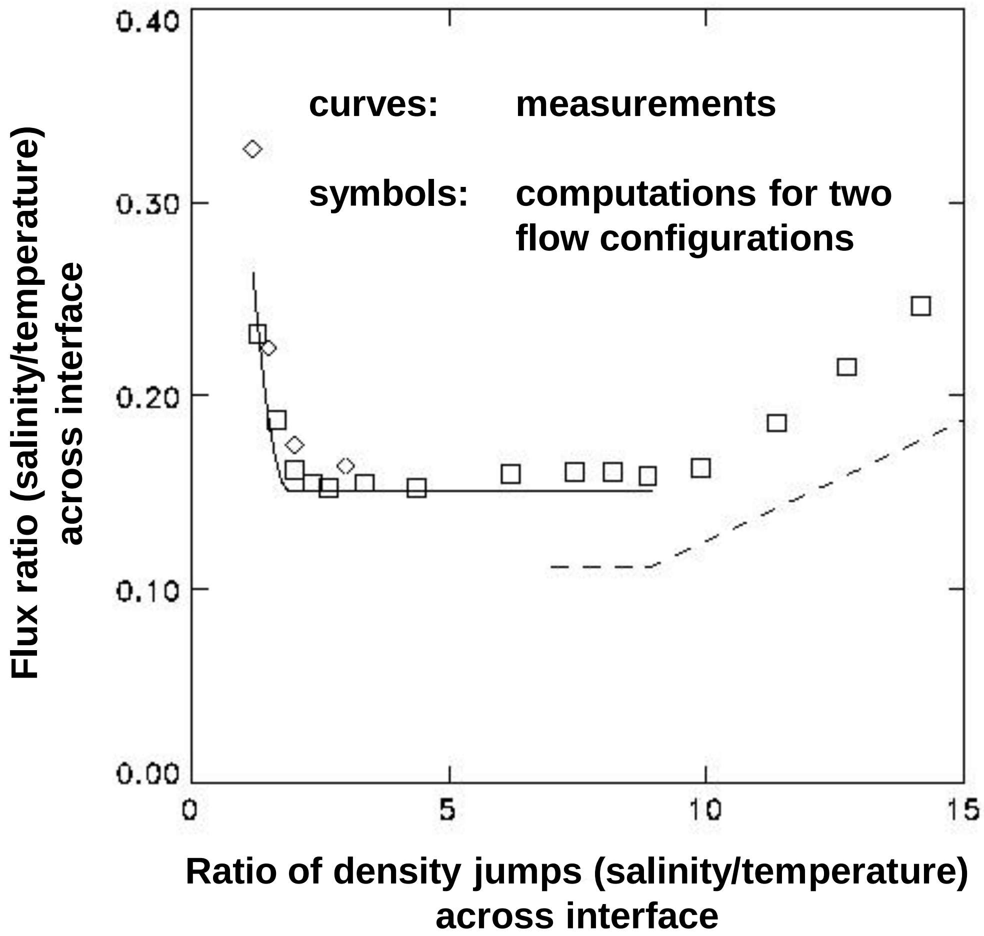
An ODT formulation requiring no parameter adjustment is compared to measurements



ODT captures the observed regimes of diffusive interface structure



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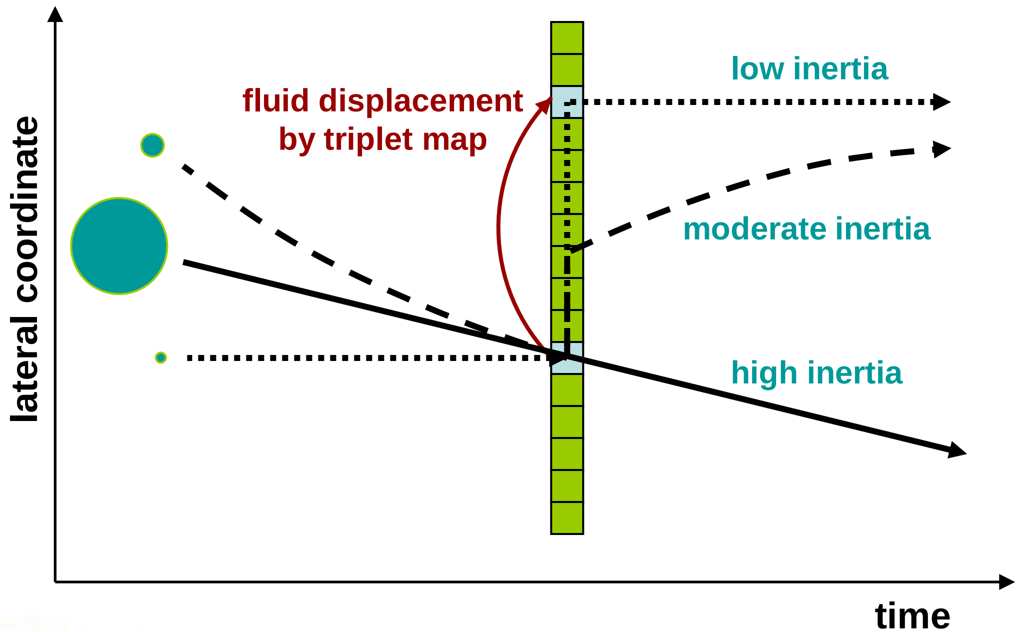
A particle-eddy interaction couples entrained particles to fluid motion (one-way coupling)

- In ODT, motion and velocity are distinct, though dynamically consistent
- Particles respond, via drag law, to motion (in ODT, eddy events)
- Because ODT eddies are instantaneous
  - an internal (eddy) time coordinate for particle-eddy interaction is introduced
  - this involves another free parameter, relating the interaction time to t

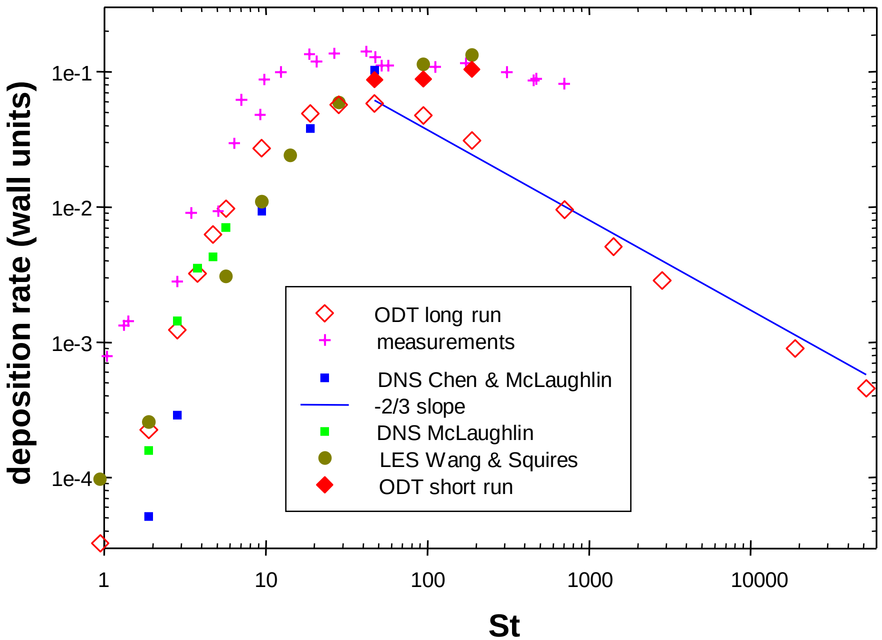


- **Eddy-time integration determines a trajectory 'jump condition' representing the eddy-induced trajectory change, adjusted so future motion is not double-counted**
- **Ballistic motion remains linear**
- **Zero-inertia (no-slip) particles follow the fluid**
- **Particle-fluid relative motion is realistic, though absolute motion is discontinuous**

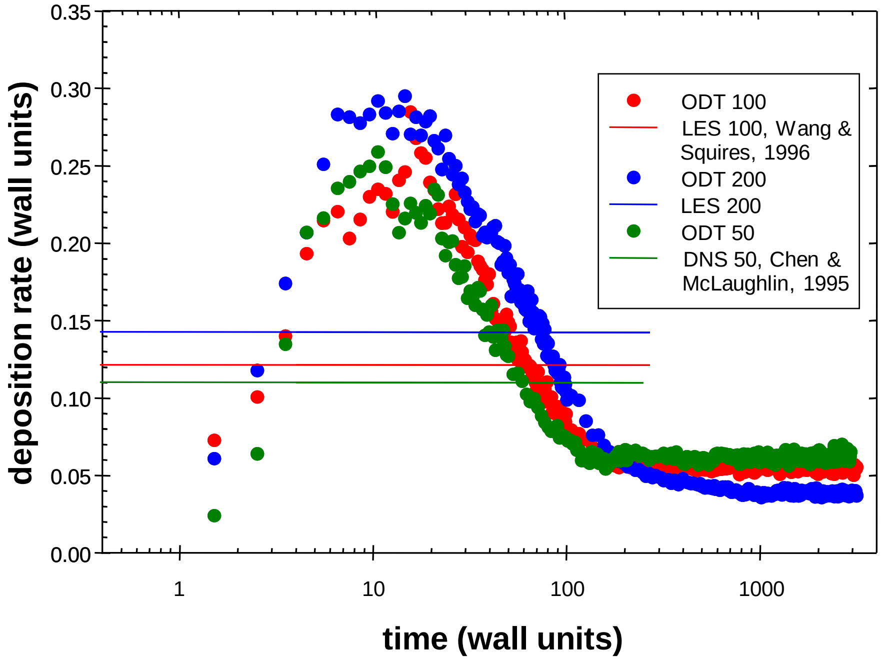
Measured and 3D-simulated wall deposition is reproduced, and a new regime is found

Wall deposition in turbulent channel flow

Dependence on Stokes number



Time variation of deposition rate (transient relaxation)



Comparisons suggest that measurements and 3D simulations are seeing initial transients rather than the late-time regime indicated by ODT

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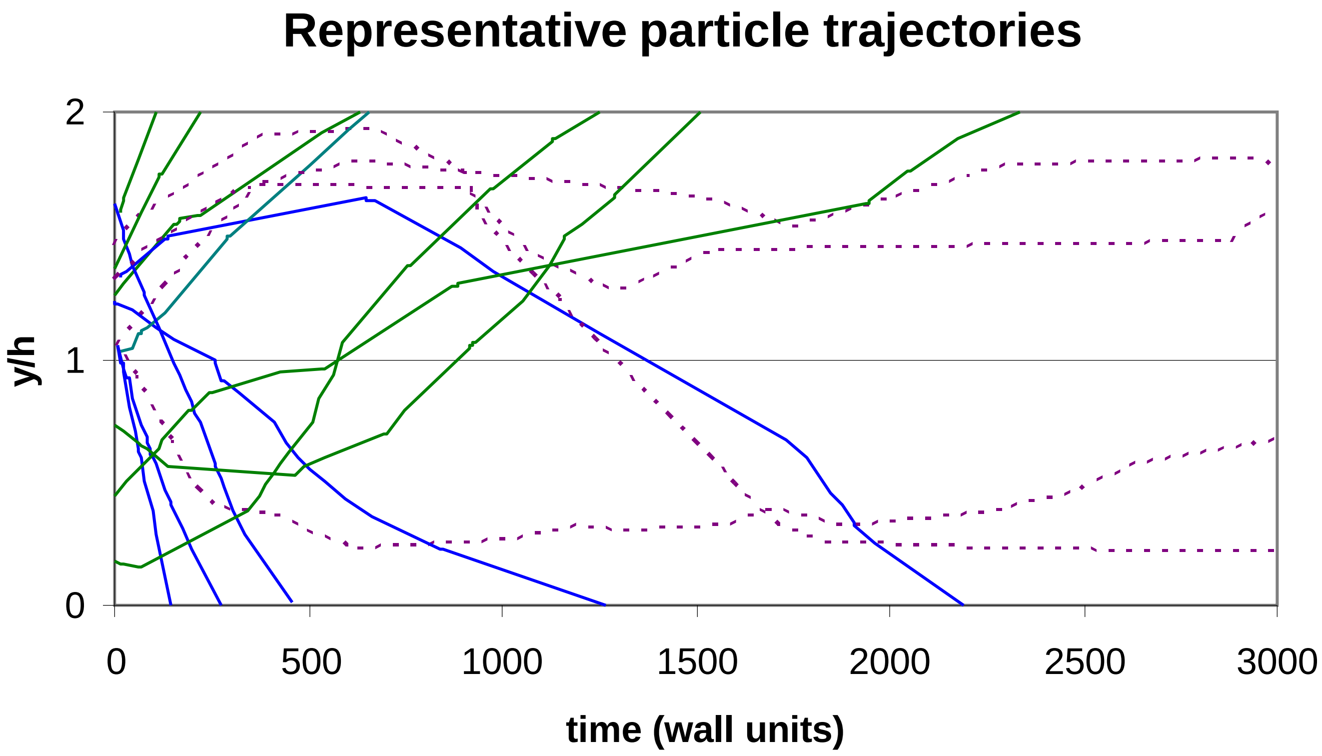
Early deposition is ballistic, late deposition is Stokes-number dependent



The -2/3 power dependence on St is explained by a simple scaling analysis. Closure analysis gives a much milder decline – and is 'validated' by data that mainly reflects initial conditions!

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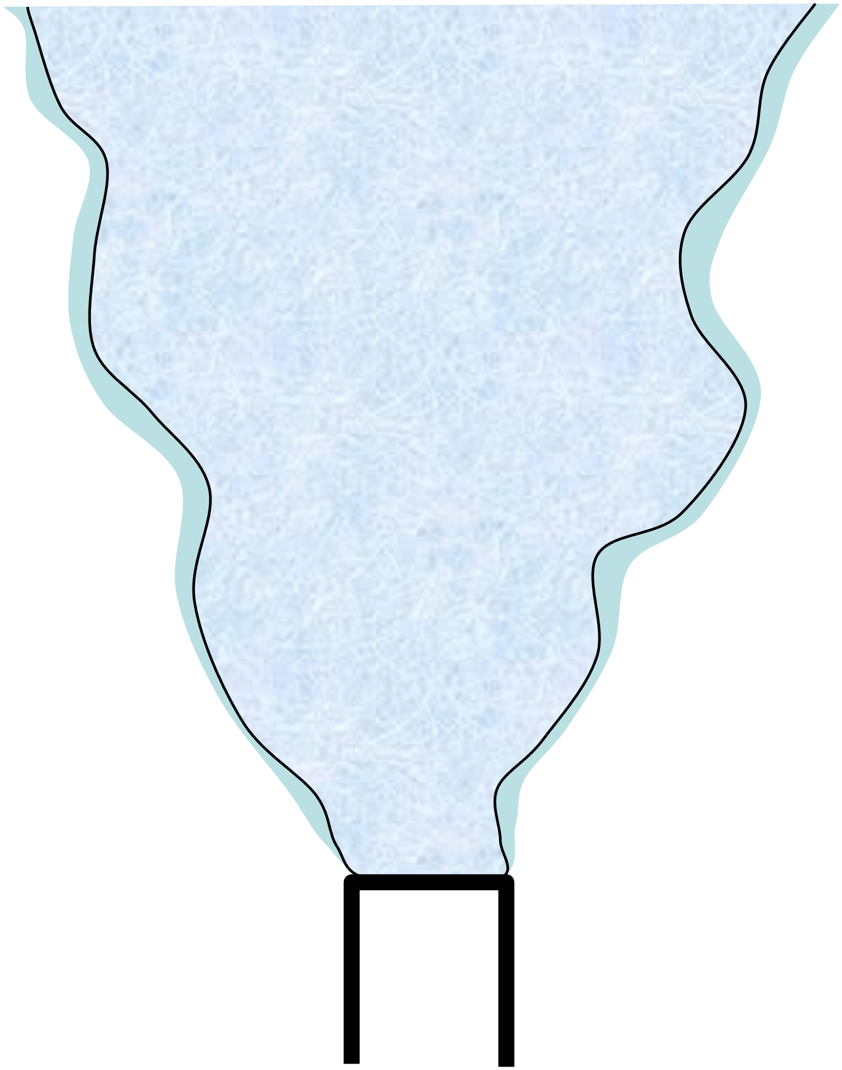
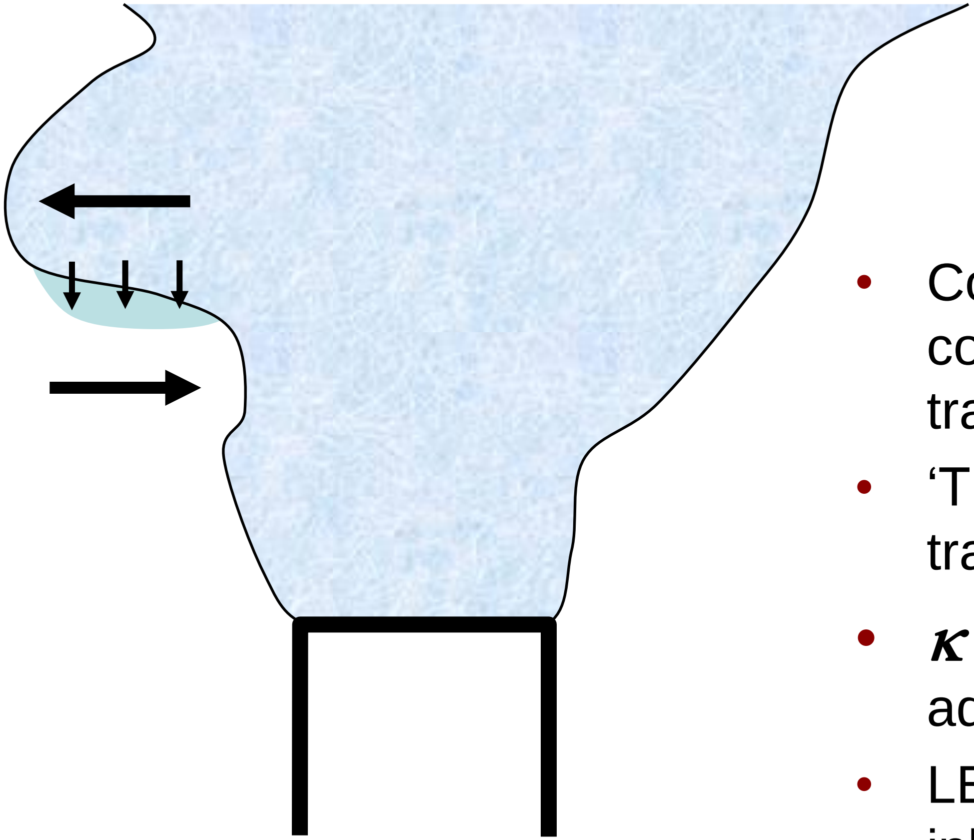
In a two-species turbulent jet, does the lighter or heavier species spread faster?



Measurements showed slower spread of the lighter species, then Saffman (1960) explained it



Key points:

- Continuity causes spatially correlated motion that affects transport
- ‘Turbulent transport’ and molecular transport are qualitatively different
- κ and κ_e are not necessarily additive, as commonly assumed
- LEM and ODT do not capture this inherently multidimensional effect

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More surprises await us

- There is much remaining to be discovered through further exploration of all turbulent flow regimes by every possible means
- LEM and ODT can make contributions that are complementary to experiments and multi-dimensional simulations
- Practical as well as fundamental scientific insights will be gained through this exploration