Toward incorporating organized eddy structures in the modelling of wall-bounded turbulence

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Outline

• Introduction

• Review of Townsend/Perry view of eddies and turbulence

• Approaches for using organized eddy concepts for modelling

• Inner and outer region interactions - prospects for modelling
Turbulent Boundary Layer

(Flow visualization using Al flakes in water channel: Cantwell et al)

Flow direction

\[ U^+ = \frac{U}{U_\tau}; \quad z^+ = \frac{zU_\tau}{\nu} \]
\[
\frac{\partial U}{\partial x} + \frac{\partial W}{\partial z} = 0
\]

\[
U \frac{\partial U}{\partial x} + W \frac{\partial U}{\partial z} = -\frac{1}{\rho} \frac{dp_1}{dx} + \frac{1}{\rho} \frac{\partial \tau}{\partial z},
\]

\[
\frac{\tau}{\rho} = -u_1 u_3 + \nu \frac{\partial U}{\partial z}, \quad \frac{\tau}{\rho} = \left( \nu_T + \nu \right) \frac{\partial U}{\partial z}
\]

\[
\nu_T = -u_1 u_3 \left( \frac{\partial U}{\partial z} \right)^{-1}
\]
\[ \nu_T = -u_1 u_3 \left( \frac{\partial U}{\partial z} \right)^{-1} \]

\[ \nu = \frac{\tau_L}{\rho} \left( \frac{\partial U}{\partial z} \right)^{-1} \]
\[ \nu_T = -\bar{u}_1 \bar{u}_3 \left( \frac{\partial U}{\partial z} \right)^{-1} \]

\[ \nu = \frac{\tau_L}{\rho} \left( \frac{\partial U}{\partial z} \right)^{-1} \]

\[ \frac{\nu_T}{(\delta U_\tau)} = \text{constant} \]
\[ \nu_T = -u_1u_3 \left( \frac{\partial U}{\partial z} \right)^{-1} \]

\[ \nu = \frac{\tau_L}{\rho} \left( \frac{\partial U}{\partial z} \right)^{-1} \]

\[ \frac{\nu_T}{(\delta U_\tau)} = \text{constant} \]

\[ \frac{\nu_T}{(\delta U_\tau)} = \phi \left( \frac{z}{\delta} \right) \]
\[ \nu_T = -u_1 u_3 \left( \frac{\partial U}{\partial z} \right)^{-1} \]
\[ \nu = \frac{\tau_L}{\rho} \left( \frac{\partial U}{\partial z} \right)^{-1} \]

\[ \frac{\nu_T}{(\delta U_\tau)} = \text{constant} \]

\[ \frac{\nu_T}{(\delta U_\tau)} = \phi \left( \frac{z}{\delta} \right) \]

\[ \frac{\nu_T}{(\delta U_\tau)} = \phi \left( \frac{z}{\delta}, \Pi \right) \]
\[ \nu_T = -u_1 u_3 \left( \frac{\partial U}{\partial z} \right)^{-1} \]

\[ \nu = \frac{\tau_L}{\rho} \left( \frac{\partial U}{\partial z} \right)^{-1} \]

\[ \frac{\nu_T}{(\delta U_\tau)} = \phi \left( \frac{z}{\delta}, \Pi \right) \]

\[ U^+ = f(z^+) + \Pi \ g(z/\delta) \]
\[ \nu_T = -u_1 u_3 \left( \frac{\partial U}{\partial z} \right)^{-1} \quad \nu = \frac{\tau_L}{\rho} \left( \frac{\partial U}{\partial z} \right)^{-1} \]

\[ \frac{\nu_T}{(\delta U_\tau)} = \phi \left( z/\delta, \Pi \right) \]

\[ (U_1 - U)^+ = f(z/\delta, \Pi) \]
\[ \nu_T = -u_1 u_3 \left( \frac{\partial U}{\partial z} \right)^{-1} \]
\[ \nu = \frac{\tau_L}{\rho} \left( \frac{\partial U}{\partial z} \right)^{-1} \]

\[ \frac{\nu_T}{(\delta U_{\tau})} = \phi \left( z/\delta, \Pi \right) \]

\[ (U_1 - U)^+ = f \left( z/\delta, \Pi \right) \]

\[ \frac{\nu_T}{(\delta U_{\tau})} = \phi \left( z/\delta, \Pi, \gamma \right) \]
\[
U \frac{\partial U}{\partial x} + W \frac{\partial U}{\partial z} = -\frac{1}{\rho} \frac{dp_1}{dx} + \frac{1}{\rho} \frac{\partial \tau}{\partial z},
\]

\[
\frac{\partial U}{\partial x} + \frac{\partial W}{\partial z} = 0
\]

\[
\frac{\tau}{\rho} = -\bar{u}_1 u_3 + v \frac{\partial U}{\partial z}
\]

\[
(U_1 - U)^+ = f(z/\delta, \Pi)
\]

\[
\frac{\tau}{\tau_0} = f_1[\eta, \Pi, S] + g_1[\eta, \Pi, S] \zeta + g_2[\eta, \Pi, S] \beta
\]

\[
\Pi, \quad S = \frac{U_1}{U_\tau}, \quad \beta = \frac{\delta^* dp}{\tau_0 dx}, \quad \zeta = S \delta \frac{d\Pi}{dx}.
\]
\[ \nu_T = -u_1 u_3 \left( \frac{\partial U}{\partial z} \right)^{-1} \]
\[ \nu = \frac{\tau_L}{\rho} \left( \frac{\partial U}{\partial z} \right)^{-1} \]
\[ (U_1 - U)^+ = f(z/\delta, \Pi) \]
\[ \frac{\nu_T}{(\delta U_\tau)} = \phi(z/\delta, \Pi, \beta, S, \zeta) \]
Non-local effects important
Outer wake region
\( z/\delta = 0.5 \)

Log region
\( z^+ = 92 \)

Ganapathisubramani et al (2005)
Organized Eddies/Coherent Structures and Turbulence Modelling

Alan Townsend

Tony Perry
Townsend (1987):

“Local” descriptions of turbulent flows are conceptually unsatisfactory when eddies controlling levels of transport extend over the entire flow width.


Rather than using exchange coefficients related to local flow variables, the layers should be looked at as an “integrated whole” with the transport properties at one point being related to motions in regions remote from the point of interest.
Townsend (1987):

Instead of assuming some form of similarity of the turbulent motion - eg. constancy of stress-intensity ratio - in all flows, the additional and varying contributions to Reynolds stresses from the organized eddies could be included and lead to better description.

Improvements in the performance of schemes for flow calculation can be made in a more rational manner from a knowledge of the organized eddies that control the flow than from empirical adjustments based on comparison of predicted and observed values of the flow parameters.

An appreciation of the mechanisms that lead to the differences in form and function of the organized eddies suggest limits to the applicability of a particular scheme to flows outside its design range.
Case study: Attached eddy model

Aim: Construct turbulence statistics given the mean-velocity field
Applying Biot-Savart integral for representative eddy (with image in wall), and using Campbell’s theorem:

\[
\frac{U_0}{\delta} = \frac{K}{2\pi \delta^2} = \frac{\pi r_0^2 \Omega_0}{2\pi \delta^2} = \frac{1}{2} q^2 \Omega_0
\]

**Figure 2.** Sketch of a representative attached eddy.

Representative eddy cross-stream vorticity distribution

Townsend eddy intensity functions
Hierarchy of eddy scales
\[
\frac{dU / U_\tau}{dz} = \int_{\delta_1}^{\delta_c} f[z / \delta] Q[\delta / \delta_c] D[\delta / \delta_c] \frac{1}{\delta^2} \, d\delta
\]

\[
\frac{\overline{u_i u_j}}{U_\tau^2} = \int_{\delta_1}^{\delta_c} I_{ij}[z / \delta] Q^2[\delta / \delta_c] D[\delta / \delta_c] \frac{1}{\delta} \, d\delta
\]

\[D[\delta / \delta_c]\] : Measure of how the p.d.f. of eddy scales departs from a -1 power law (geometric progression)

\[Q[\delta / \delta_c]\] : Measure of velocity scale variation across hierarchies
\[ \frac{dU_D^*}{d\lambda_E} = \int_{-\infty}^{\infty} h[\lambda] e^{-\lambda} T[\lambda - \lambda_E] w[\lambda - \lambda_E] \, d\lambda \]

\[ \frac{u_i u_j}{U_t^2} = \int_{-\infty}^{\infty} J_{ij}[\lambda] T^2[\lambda - \lambda_E] w[\lambda - \lambda_E] \, d\lambda \]

\[ \lambda = \log[\delta/z], \quad \lambda_E = \log[\delta_c/z], \quad \lambda_1 = \log[\delta_1/z] \]

\[ w[\lambda - \lambda_E] = D[\delta/\delta_c], \quad T[\lambda - \lambda_E] = Q[\delta/\delta_c], \quad h[\lambda] = f[z/\delta] \]

\[ J_{ij}[\lambda] = I_{ij}[z/\delta] \]
Similarly for spectra:

\[
\frac{\Phi_{ij}[k_1 z, z/\delta_c, z/\delta_1]}{U_t^2} = \int_{\delta_1}^{\delta_c} G_{ij}[k_1 z, z/\delta] Q^2[\delta/\delta_c] D[\delta/\delta_c] \frac{1}{\delta} \, d\delta
\]

\[
\frac{\Psi_{ij}[\alpha_z, \lambda_E, \lambda_1]}{U_t^2} = \int_{-\infty}^{\infty} g_{ij}[\alpha_z, \lambda] T^2[\lambda - \lambda_E] w[\lambda - \lambda_E] \, d\lambda,
\]

\[
\Psi_{ij}[\alpha_z, \lambda_E, \lambda_1] = k_1 z \Phi_{ij}[k_1 z, z/\delta_c, z/\delta_1],
\]

\[
g_{ij}[\alpha_z, \lambda] = k_1 z G_{ij}[k_1 z, z/\delta]
\]
Compute turbulence statistics (Reynolds stresses, spectra etc), given

- Mean-velocity flow field
- Equations of motion
\[
\frac{\nu_T}{(\delta U_T)} = \frac{\int_{-\infty}^{\infty} J_{13}(\lambda) T^2(\lambda - \lambda_E) w(\lambda - \lambda_E) \, d\lambda}{\int_{-\infty}^{\infty} h(\lambda) e^{-\lambda} T(\lambda - \lambda_E) w(\lambda - \lambda_E) \, d\lambda}
\]
\[
\frac{\nu_T}{(\delta U_\tau)} = \frac{\int_{-\infty}^{\infty} J_{13}(\lambda) T^2(\lambda - \lambda_E)w(\lambda - \lambda_E) \, d\lambda}{\int_{-\infty}^{\infty} h(\lambda)e^{-\lambda} T(\lambda - \lambda_E)w(\lambda - \lambda_E) \, d\lambda}
\]

\[
\Pi, S, \zeta, \beta \quad \text{from experiment}
\]

\[
-\frac{\bar{u}_1 \bar{u}_3}{U_t^2} = f_1[\eta, \Pi, S] + g_1[\eta, \Pi, S] \zeta + g_2[\eta, \Pi, S] \beta
\]

\[
-\frac{\bar{u}_1 \bar{u}_3}{U_t^2} = \int_{-\infty}^{\infty} J_{13}[\lambda] T^2(\lambda - \lambda_E)w(\lambda - \lambda_E) \, d\lambda
\]

\[
\Pi
\]

\[
\frac{dU^*_D}{d\lambda_E} = \int_{-\infty}^{\infty} h[\lambda]e^{-\lambda} T[\lambda - \lambda_E]w[\lambda - \lambda_E] \, d\lambda,
\]

\[
\frac{dU^*_D}{d\lambda_E} = T^2 \omega
\]

\[
\frac{dU^*_D}{d\lambda_E} = T \omega
\]
\[ \frac{\nu_T}{(\delta U_\tau)} = \frac{\int_{-\infty}^{\infty} J_{13}(\lambda) T^2(\lambda - \lambda_E)w(\lambda - \lambda_E) \, d\lambda}{\int_{-\infty}^{\infty} h(\lambda)e^{-\lambda} T(\lambda - \lambda_E)w(\lambda - \lambda_E) \, d\lambda} \]

Require $T=1$ (same velocity scale for each hierarchy) for

\[ (U_1 - U)^+ = f(z/\delta, \Pi), \quad \frac{\nu_T}{(\delta U_\tau)} = \phi(z/\delta, \Pi) \]

⇒ works for quasi-equilibrium boundary layers, self-similar jets etc
Coflowing jets

*T. B. Nickels and A. E. Perry*

**Figure 20.** Reynolds stresses from experiment ($x/D = 30, \lambda = 2$) compared with (solid lines).

*T. B. Nickels and I. Marusic*
Attached eddy model calculation road map

\[ \Pi, S, \zeta, \beta \]
from experiment

\[ - \frac{u_1 u_2}{U^2} = f_1[\eta, \Pi, S] + g_1[\eta, \Pi, S] \zeta + g_2[\eta, \Pi, S] \beta \]

\[ \frac{u_1 u_2}{U^2} = \int_{-\infty}^{\infty} J_{11}[\lambda] T^2[\lambda - \lambda_E] \omega[\lambda - \lambda_E] d\lambda \]

\[ T^2 \omega \text{ by deconvolution} \]

\[ \frac{u_1^2}{U^2} = \int_{-\infty}^{\infty} J_{11}[\lambda] T^2[\lambda - \lambda_E] \omega[\lambda - \lambda_E] d\lambda \]

\[ \frac{u_2^2}{U^2} \text{ and similarly } \frac{u_1^2}{U^2} \text{ and } \frac{u_3^2}{U^2} \text{ by convolution} \]
Wall-wake attached eddy model of wall turbulence - with and without pressure gradients

cf. Coles’ law of the wall, law of the wake
For equilibrium sink flow

$$\left( \frac{u_1 u_3}{U_1^2} \right)_A = 1 - \eta + \eta \log [\eta]$$

$$\left( \frac{u_1 u_3}{U_1^2} \right)_A = \int_{-\infty}^{\infty} f_{13}[\lambda] T^2_A[\lambda - \lambda_3] \omega_A[\lambda - \lambda_3] \ d\lambda$$

$$\left( \frac{u_1 u_3}{U_1^2} \right)_A \text{ by deconvolution}$$

$$\left( \frac{u_1 u_3}{U_1^2} \right)_B = \int_{-\infty}^{\infty} f_{13}[\lambda] T^2_B[\lambda - \lambda_3] \omega_B[\lambda - \lambda_3] \ d\lambda$$

$$\left( \frac{u_1 u_3}{U_1^2} \right)_B \text{ by deconvolution}$$

Composite profiles

Figure 18. Overview of calculations for the wall-wake eddy structure model.
Attached Eddy Model: wall and wake eddies
Limitations of single-uncorrelated eddy paradigm
Thus far: assuming statistically uncorrelated eddies
Packets: Adrian, Meinhart and Tomkins (2000)

Christensen & Adrian (2001)
Tomkins & Adrian (2003)
Simultaneous dual-plane PIV

Conditional Average
Conditioned on swirl event at $z^+=200$

Hambleton, Hutchins & Marusic (2006)
\[
\frac{U_A^{(n)}}{U_0} = f_A(\frac{x}{\delta}, \frac{y}{\delta}, \frac{z_A}{\delta}), \quad \frac{U_B^{(n)}}{U_0} = f_B(\frac{x}{\delta}, \frac{y}{\delta}, \frac{z_B}{\delta})
\]

\[
\psi_A(k_1 \delta, \frac{y}{\delta}, \frac{z_A}{\delta}) = \frac{1}{N} \sum_{n=1}^{N} \left\{ F_A^{(n)}(k_1 \delta, \frac{y}{\delta}, \frac{z_A}{\delta}) e^{-ik_1 \lambda x^{(n)}_0} \right\}
\]

\[
\psi_B(k_1 \delta, \frac{y}{\delta}, \frac{z_B}{\delta}) = \frac{1}{N} \sum_{n=1}^{N} \left\{ F_B^{(n)}(k_1 \delta, \frac{y}{\delta}, \frac{z_B}{\delta}) e^{-ik_1 \lambda x^{(n)}_0} \right\}
\]

\[
\Phi_{AB}(k_1 \delta_c) = \int_{\delta_1}^{\delta_c} \left\{ \frac{U_0^2}{\lambda_x \lambda_y} \int_{-\infty}^{\infty} \psi_A^* \psi_B \ d(\frac{y}{\delta}) \right\} Q^2(\frac{\delta}{\delta_c}) D(\frac{\delta}{\delta_c}) \frac{1}{\delta} \ d\delta
\]

\[
R_{AB}(\Delta x / \delta_c) = \int_{-\infty}^{\infty} \Phi_{AB}(k_1 \delta_c) e^{ik_1 \delta_c \Delta x / \delta_c} \ d(k_1 \delta_c)
\]
Non-local effects important

- interactions beyond linear superposition
Log layer structures and interaction across the boundary layer

Superstructures/VLSM
High frame-rate PIV
Dennis & Nickels (2011)
Conditional average on \(-u\) at \(\Delta x=0, \Delta y=0, z/\delta = 0.036\)

ASL - Utah
Superstructures associated with “outer-peak” in spectra

\[ \lambda_x^+ \approx 1000 \]
\[ \lambda_x / \delta \approx 6 \delta \]
\[ z^+ \approx 15 \]
\[ z = 0.06 \delta \]
\[ \frac{\overline{u^2}}{U^2} \]

\[ Re_\tau \approx 7300 \]
hot-wire data \( l^+ \approx 22 \)
Superstructure influence near the wall: modulation & superposition
Simultaneous velocities at inner and outer peak locations

\[ \frac{z}{\delta} = 0.06 \]

\[ \frac{x}{\delta} = 15 \]

\[ \text{Re}_{\tau} = 7300 \]
Simultaneous velocities at inner and outer peak locations

$z/\delta = 0.06$

$z^+ = 15$

$Re_T = 7300$

$R = 0.72$ with 14 deg. shift applied
Simultaneous velocities at inner and outer peak locations

Evidence of amplitude modulation

$z/\delta = 0.06$

$z^+ = 15$

$Re_\tau = 7300$
Superstructure interaction in near-wall region

$(a)$ $u^+$ streamwise

$(b)$ $w^+$ wall-normal

$(c)$ $v^+$ spanwise

$(d)$ $-uw^+$ Reynolds shear stress

$Re_\tau = 934$ DNS data at $z^+ = 15$
Amplitude modulation in near-wall region

\[ Re_\tau = 934 \text{ DNS data at } z^+ = 15 \]
Marusic, Mathis & Hutchins (2010), *Science*. vol. 329
Quantifying superstructure interaction on near-wall region:

Can we accurately model (predict) near-wall signal from log-region signature?
Challenge of accessing near-wall region at high Re

Princeton superpipe measurements

Morrison et al. (2004)
Near-wall models for large-eddy simulation

Chapman (1979):
LES - outer layer resolution $\sim \text{Re}^{0.4}$
- wall layer resolution $\sim \text{Re}^{1.8}$

Resolved wall region in LES, typically:

\[
\frac{\Delta z_{\text{min}}}{\delta} \approx \frac{1}{\text{Re}_\tau}, \quad \frac{\Delta x}{\delta} \approx \frac{100}{\text{Re}_\tau}, \quad \frac{\Delta y}{\delta} \approx \frac{50}{\text{Re}_\tau}
\]

$\text{Re}_\tau = 10^6$: wall-layer resolved requires 99% of grid points to resolve first 10% of boundary layer
Mind the gap: a guideline for large eddy simulation

BY WILLIAM K. GEORGE¹,* AND MURAT TUTKUN²

Phil. Trans. R. Soc. A (2009) 367, 2839–2847

Based on measurements of correlations across the boundary layer:

“This suggests that it might be possible to build a near-wall model that is in sync with the outer flow (i.e. follows it), perhaps quite independent of considerations such as Reynolds number and spectral gaps. Note that this is quite the opposite of the prevailing view for the last 40 years or so that it is the wall region (with its streaks and bursts) that drives the outer flow. In fact, it assumes the opposite: namely that the inner flow is driven by the outer.”
\[
\tilde{u}^+ (z^+) = u^* (z^+) \left[ 1 + \beta u_{LS}^+ (z_O^+, \theta_{LS}) \right] + \alpha u_{LS}^+ (z_O^+, \theta_{LS})
\]
with \(LS\) for Large-Scale
keep energy \(\lambda_x^+ > 7000\)
\[ \tilde{u}^+ (z^+) = u^* (z^+) \left[ 1 + \beta u_{LS}^+ (z_O^+, \theta_{LS}) + \alpha u_{LS}^+ (z_O^+, \theta_{LS}) \right] \]

The measured outer large-scale signal

Universal signal

Predicted near-wall signatures

Amplitude modulate \( u^* \)

Linear superposition of large-scales

The measured outer large-scale signal

The measured outer large-scale signal
Experimental setup for calibration of the model

Two-point hot-wire simultaneous measurements:

\( Re_\tau = 7300 \) (arbitrary chosen Reynolds number)
\( U_\infty = 10.02 \, \text{m/s} \)
\( U_\tau = 0.34 \, \text{m/s} \)
\( \delta = 0.33 \)
\( l^+ = 22 \)

\( z_O^+ = 3.9 Re_\tau^{1/2} \)

\( 6.28 < z^+ < 303 \)
Experimental setup for calibration of the model

Two-point hot-wire simultaneous measurements:
\( Re_T = 7300 \) (arbitrary chosen Reynolds number)
\( U_\infty = 10.02 \text{ m/s} \)
\( U_T = 0.34 \text{ m/s} \)
\( \delta = 0.33 \)
\( l^+ = 22 \)
Experimental setup for calibration of the model

Two-point hot-wire simultaneous measurements:

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\( U_\tau = 0.34 \text{ m/s} \)

\( \delta = 0.33 \)

\( l^+ = 22 \)

\[ z_O^+ = 3.9 \text{Re}_{\tau}^{1/2} \]

\[ 6.28 < z^+ < 303 \]

\[ \frac{\text{u}^2}{U_\tau^2} \]

Marusic et al. (University of Melbourne)

APS 2010 6 / 14
Experimental setup for calibration of the model

Two-point hot-wire simultaneous measurements:
\( Re_\tau = 7300 \) (arbitrary chosen Reynolds number)
\( U_\infty = 10.02 \text{ m/s} \)
\( U_\tau = 0.34 \text{ m/s} \)
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\[ z_O^+ = 3.9 Re_\tau^{1/2} \]

6.28 < \( z^+ \) < 303
Experimental setup for calibration of the model

Two-point hot-wire simultaneous measurements:
$Re \tau = 7300$ (arbitrary chosen Reynolds number)
$U_\infty = 10.02$ m/s
$U_\tau = 0.34$ m/s
$\delta = 0.33$
$l^+ = 22$

$U_O^+$ Outer probe (fixed)
$6.28 < z^+ < 303$
$z_O^+ = 3.9 Re_\tau^{1/2}$

$U^+$ Inner probe (moving)

$\frac{\overline{u'^2}}{U^2}$

$z^+$
Determination of the model's parameters

\[ u^+(z^+) = u^*(z^+) \left[ 1 + \beta \ u^+_{LS} \left( z^+_O, \theta_{LS} \right) \right] + \alpha \ u^+_{LS} \left( z^+_O, \theta_{LS} \right) \]

calibration measurement

\[ z^+_O = 3.9 Re_{\tau}^{1/2} \]

6.28 < z⁺ < 303
Determination of the model’s parameters

\[ u^+(z^+) = u^*(z^+) \left[ 1 + \beta \ u_{LS}^+(z_O^+, \theta_{LS}) \right] + \alpha \ u_{LS}^+(z_O^+, \theta_{LS}) \]

\( u^+(z^+) \) and \( u_{LS}^+(z_O^+) \) known from the 2 points measurements
Determination of the model’s parameters

\[ u^+ (z^+) = u^* (z^+) \left[ 1 + \beta \ u^+_L (z^+_O, \theta_{LS}) \right] + \alpha \ u^+_L (z^+_O, \theta_{LS}) \]

\( u^+ (z^+) \) and \( u^+_L (z^+_O) \) known from the 2 points measurements

\( u^* (z^+) \), \( \alpha (z^+) \), \( \beta (z^+) \) and \( \theta_{LS} (z^+) \) need to be determined

calibration measurement

\( z^+_O = 3.9 Re^{1/2} \)

6.28 < \( z^+ \) < 303
Determination of the model’s parameters

\[ u^+(z^+) = u^*(z^+) \left[ 1 + \beta u_{LS}^+(z_O^+, \theta_{LS}) \right] + \alpha u_{LS}^+(z_O^+, \theta_{LS}) \]

\[ \alpha(z^+) = \max \left( R u_{LS}^+(z^+), u_{LS}^+(z_O^+) \right) \text{ and corresponding } \theta_{LS}(z^+) \]

calibration measurement

\[ z_O^+ = 3.9 Re^{1/2} \]

\[ 6.28 < z^+ < 303 \]
Determination of the model’s parameters

\[ u^+(z^+) = u^*(z^+) \left[ 1 + \beta \ u_{LS}^+(z_O^+, \theta_{LS}) \right] + \alpha \ u_{LS}^+(z_O^+, \theta_{LS}) \]

\[ \alpha(z^+) = \max \left( R_{u_{LS}(z^+)} u_{LS}^+(z_O^+) \right) \text{ and corresponding } \theta_{LS}(z^+) \]
Determination of the model’s parameters

\[ u^+(z^+) = u^*(z^+) \left[ 1 + \beta \ u_{LS}^\pm (z_O^+, \theta_{LS}) \right] + \alpha \ u_{LS}^\pm (z_O^+, \theta_{LS}) \]

\[ \alpha(z^+) = \max \left( R_{u_{LS}(z^+) u_{LS}(z_O^+)} \right) \] and corresponding \( \theta_{LS}(z^+) \)

calibration measurement

\[ z_O^+ = 3.9 Re_1^{1/2} \]

\[ 6.28 < z^+ < 303 \]

\[ \Delta z \text{ decreasing} \]

\[ \Delta x/\delta \]
Determination of the model's parameters

\[ u^+(z^+) = u^*(z^+) \left[ 1 + \beta \ u_{LS}^+(z_O^+, \theta_{LS}) \right] + \alpha \ u_{LS}^+(z_O^+, \theta_{LS}) \]

\[ \alpha(z^+) = \max \left( R u_{LS}^+(z^+) u_{LS}^+(z_O^+) \right) \]

and corresponding \( \theta_{LS}(z^+) \)

\[
\begin{align*}
U_O^+ & \quad \text{Outer probe} \\
U^+ & \quad \text{Inner probe} \\
z_O^+ &= 3.9 Re^{1/2} \\
6.28 < z^+ < 303
\end{align*}
\]
Determination of the model's parameters

\[ u^+(z^+) = u^*(z^+) \left[ 1 + \beta \ u_{LS}^+(z_O^+, \theta_{LS}) \right] + \alpha \ u_{LS}^+(z_O^+, \theta_{LS}) \]

\[ \alpha(z^+) = \max \left( R_{u_{LS}(z^+)u_{LS}(z_O^+)} \right) \text{ and corresponding } \theta_{LS}(z^+) \]

Calibration measurement

\[ z_O^+ = 3.9 Re^{1/2} \]

\[ 6.28 < z^+ < 303 \]
Determination of the model's parameters

\[ u^+(z^+) = u^*(z^+) \left[ 1 + \beta u_{LS}^+ (z_O^+, \theta_{LS}) \right] + \alpha u_{LS}^+ (z_O^+, \theta_{LS}) \]

\[ u^* (z^+) = \frac{u^+ (z^+) - \alpha u_{LS}^+ (z_O^+, \theta_{LS})}{1 + \beta u_{LS}^+ (z_O^+, \theta_{LS})}, \quad \beta \text{ such as } AM(u^*) = 0 \]

calibration measurement

\[ z_O^+ = 3.9 Re^{1/2} \]

\[ 6.28 < z^+ < 303 \]
Pre-multiplied energy spectra map reconstruction

\[ \tilde{u}^+(z^+) = u^*(z^+) \left[ 1 + \beta u_{LS}^+(z_O^+, \theta_{LS}) \right] + \alpha u_{LS}^+(z_O^+, \theta_{LS}) \]

Single-point hot-wire measurements:
\[ Re_\tau = 2800 \]
\[ U_\infty = 11.97 \text{ m/s} \]
\[ U_\tau = 0.44 \text{ m/s} \]
\[ \delta = 0.01 \text{ m} \]
\[ l^+ = 22 \]

\[ U_O^+ \quad \text{Outer probe} \quad z_O^+ = 3.9 Re_\tau^{1/2} \]
Pre-multiplied energy spectra map reconstruction

$$\tilde{u}^+(z^+) = u^*(z^+) \left[ 1 + \beta u_{LS}^+(z_O^+, \theta_{LS}) \right] + \alpha u_{LS}^+(z_O^+, \theta_{LS})$$

Single-point hot-wire measurements:

$Re_\tau = 2800$

$U_\infty = 11.97 \text{ m/s}$

$U_\tau = 0.44 \text{ m/s}$

$\delta = 0.01 \text{ m}$

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\( l^+ = 22 \)

\( z_O^+ = 3.9 Re_\tau^{1/2} \)
Pre-multiplied energy spectra map reconstruction

\[ \tilde{u}^+(z^+) = u^*(z^+)\left[1 + \beta u_{LS}^+(z_O^+, \theta_{LS})\right] + \alpha u_{LS}^+(z_O^+, \theta_{LS}) \]

Single-point hot-wire measurements:

- \( Re_\tau = 3900 \)
- \( U_\infty = 11.87 \text{ m/s} \)
- \( U_\tau = 0.43 \text{ m/s} \)
- \( \delta = 0.14 \text{ m} \)
- \( l^+ = 21 \)

\( U^+_O \) Outer probe \( z^+_O = 3.9 Re_\tau^{1/2} \)

\( \lambda_x^+ \) \( \lambda_{x/\delta} \) \( z^+/\delta \)
Pre-multiplied energy spectra map reconstruction

\[ \tilde{u}^+(z^+) = u^*(z^+) \left[ 1 + \beta u^+_LS(z_O^+, \theta_{LS}) \right] + \alpha u^+_LS(z_O^+, \theta_{LS}) \]

Single-point hot-wire measurements:

- \( Re_\tau = 7350 \)
- \( U_\infty = 10.03 \) m/s
- \( U_\tau = 0.32 \) m/s
- \( \delta = 0.33 \) m
- \( l^+ = 23 \)

\[ z_O^+ = 3.9 Re_\tau^{1/2} \]

\( Re_\tau = 7350 \)
Pre-multiplied energy spectra map reconstruction

\[ \tilde{u}^+(z^+) = u^*(z^+) \left[ 1 + \beta u^+_{LS}(z_O^+, \theta_{LS}) \right] + \alpha u^+_{LS}(z_O^+, \theta_{LS}) \]

Single-point hot-wire measurements:

- \( Re_\tau = 13600 \)
- \( U_\infty = 20.63 \text{ m/s} \)
- \( U_\tau = 0.67 \text{ m/s} \)
- \( \delta = 0.31 \text{ m} \)
- \( l^+ = 22 \)

\( U_O^+ \rightarrow \) Outer probe

\( z_O^+ = 3.9 Re_\tau^{1/2} \)

\( Re_\tau = 13600 \)

\( \lambda_x^+ \) vs. \( z^+ \) and \( \lambda_x^+ \) vs. \( z/\delta \) in the same plot with a grid.

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\( \lambda_x^+ \) vs. \( z^+ \) and \( \lambda_x^+ \) vs. \( z/\delta \) in the same plot with a grid.
Pre-multiplied energy spectra map reconstruction

\[ \tilde{u}^+(z^+) = u^*(z^+) \left[ 1 + \beta u_L^+(z_O^+, \theta_{LS}) \right] + \alpha u_L^+(z_O^+, \theta_{LS}) \]

Single-point hot-wire measurements:
- \( Re_{\tau} = 19000 \)
- \( U_\infty = 30.20 \text{ m/s} \)
- \( U_{\tau} = 0.96 \text{ m/s} \)
- \( \delta = 0.30 \text{ m} \)
- \( l^+ = 22 \)

\( z^+_O = 3.9 Re_{\tau}^{1/2} \)
Pre-multiplied energy spectra map reconstruction

\[ \tilde{u}^+(z^+) = u^*(z^+) \left[ 1 + \beta u_{LS}^+(z_O^+, \theta_{LS}) \right] + \alpha u_{LS}^+(z_O^+, \theta_{LS}) \]

Single-point sonic anemometer measurements:
\( Re_\tau = 1.4 \times 10^6 \)
\( U_\infty \) unknown
\( U_\tau = 0.26 \text{ m/s} \)
\( \delta \simeq 100 \text{ m} \)

\( z_O^+ = 3.9 Re_\tau^{1/2} \)

\( Re_\tau = 1.4 \times 10^6 \)
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Statistics

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Modeling near-wall turbulent flows

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\[ Re_\tau = 1.4 \times 10^6 \]

\[ Re_\tau = 19000 \]

\[ Re_\tau = 7300 \]

\[ Re_\tau = 2800 \]
Statistics

\[
\begin{align*}
\text{Re} & \tau = 2800 \\
\text{Re} & \tau = 7300 \\
\text{Re} & \tau = 19000 \\
\end{align*}
\]

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Statistics

$\frac{u^6}{U^6}$

$Re_\tau$ increasing

$Re_\tau = 1.4 \times 10^6$

$Re_\tau = 19000$

$Re_\tau = 7300$

$Re_\tau = 2800$

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Prediction for other wall-bounded flows

Channel, Pipe, APG-TBL and ZPG-TBL comparison

- Four different flow geometries (all in Melbourne)
  - Zero-Pressure-Gradient Turbulent Boundary Layer (ZPG-TBL)
  - Adverse-Pressure-Gradient Turbulent Boundary Layer (APG-TBL)
  - Channel
  - Pipe
- Kármán number $Re_\tau \simeq 3000 - 3500$

<table>
<thead>
<tr>
<th>Facility</th>
<th>$Re_\tau$</th>
<th>$U_\infty$ (m/s)</th>
<th>$\delta$ (m)</th>
<th>$\nu/U_\tau$ (μm)</th>
<th>$l^+$</th>
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<tbody>
<tr>
<td>ZPG-TBL</td>
<td>3020</td>
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<td>0.10</td>
<td>33.2</td>
<td>30</td>
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<tr>
<td>APG-TBL</td>
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<td>17.1</td>
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<td>32.1</td>
<td>16</td>
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<tr>
<td>Channel</td>
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<td>23.1</td>
<td>0.05</td>
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<tr>
<td>Pipe</td>
<td>3005</td>
<td>24.3</td>
<td>0.05</td>
<td>16.4</td>
<td>30</td>
</tr>
</tbody>
</table>
Prediction for other wall-bounded flows

![Graph showing predictions for different wall-bounded flows](image-url)

- Measurements
- Prediction
- Adjusted prediction

Ref.:
- ZPG-TBL
- Pipe
- Channel
- APG-TBL

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Prediction for other wall-bounded flows

Adjustment of $\beta$ and $\alpha$:

$$\tilde{u}^+(z^+) = u^*(z^+) \left[ 1 + \beta u_{LS}^+(z_O^+, \theta_{LS}) \right] + \alpha u_{LS}^+(z_O^+, \theta_{LS})$$
Prediction for other wall-bounded flows

Adjustment of $\beta$ and $\alpha$:

$$\tilde{u}^+(z^+) = u^*(z^+) \left[ 1 + \beta u_{LS}^+(z_O^+, \theta_{LS}) \right] + \alpha u_{LS}^+(z_O^+, \theta_{LS})$$
Prediction for other wall-bounded flows

Adjustment of $\beta$ and $\alpha$:

$$\tilde{u}^+(z^+) = u^*(z^+) [1 + \beta u^+_{LS}(z_O^+, \theta_{LS})] + \alpha u^+_{LS}(z_O^+, \theta_{LS})$$
Other considerations / future directions
Uniform momentum zones in wall turbulence (consistent with packets of eddies)

Modelling approaches beyond Reynolds decomposition?
Conclusions / Comments

• Coherent eddy structure concepts helpful for modelling the energy-containing, heterogeneous, motions in turbulence. But, this is still at a rudimentary stage and recent experiments (looking at 2D/3D fields) have been important in shaping our view of the what the important eddies are and how they interact across the flow.

• Superstructures or very-large-scale motions play a key role in the dynamics of wall turbulence.

• The results support the concept of a universal inner-region that is modified through a modulation and superposition of the large-scale outer motions, which are specific to the geometry or imposed streamwise pressure gradient acting on the flow. Predictive (non-linear) model based on these observations is seen to work well.

• Next step is to integrate a dynamical causality to the kinematics. Evidence of modulation is felt to be an important clue towards this aim.

• Is it worthwhile rethinking Reynolds decomposition (cf something between RANS and LES) ?