Overview

- Empirics: Stylized facts
- Microscopic and macroscopic models: typical examples:
  - Linear stability: Which concepts are relevant for describing traffic flow?
  - From the stability diagram to the “dynamic state diagram”: Mechanisms for generating the observed spatiotemporal and local phenomena
- Numerical examples: Car-following, CA and macroscopic models with one, two, or three phases …
- Conclusion: How many traffic “phases” are necessary?
Stylized facts relating to local aspects: scattered flow-density data

Dutch A9 Haarlem-Amsterdam

(a) Rottepolderplein S 17

Badroeverderp

43.3 km
42.3 km
41.8 km
41.3 km
40.8 km
39.6 km
37.6 km
36.9 km
36.6 km
35.9 km

D1 D2 D3 D4 D5 D6 D7 D8 D9

Active bottleneck

D2 D3

Flow (vehicles/h)

(c) D22 D26

Q (vehicles/h)

Nonhomogeneous-in stationary

Homogeneous-stationary

Homogeneous-in speed

German A9-South

Martin Treiber
2. Stylized facts relating to spatiotemporal data

- **Downstream front:** Fixed or moving upstream with velocity $v_g$
- **Upstream front:** Non-characteristic (pos/neg.) velocity
- **Internal structures:** Moving all with $v_g$
- **Amplitude** of internal structures grows when moving upstream
- **Frequency** grows with severity of bottleneck

A9 South (north of Munich)
2(a) The bottlenecks may be different in nature

- Uphill bottleneck (Irschenberg)
- Lane closing bottleneck
2(b): The patterns may form composite structures

- Extended, Stop&Go
- German A5-South
- Localized, moving
- Localized, fixed
2(c): To “make a jam”, one needs three ingredients …

- Three „ingredients“:
- 2. High traffic demand (inflow)
- 3. Spatial inhomogeneity („bottleneck“)
- 4. Perturbation in traffic flow
Summary: Typical spatiotemporal patterns

German A5 near Frankfurt

- Velocity (km/h)
- Location (km)
- Time (h)

MLC, TSC, OCT, PLC, HIC

Q_{lp} (vehicles/km/lane) vs. ΔQ (vehicles/km/lane)
II Stability: 1. Which types are relevant for traffic flow?

- Three kinds of linear instabilities:
  - Convective string instability,
  - Absolute string instability,
  - Absolute local instability.

- Additional nonlinear instabilities (metastability, hysteresis)

Simulate ...

\(s_1=14\ m, \ a=0.6\ m/s^2\)
2. Collective instabilities: Mathematical and numerical definitions

Linear modes:

\[ A_k(x, t) = e^{ikx} e^{\lambda(k)t} \]

Localized perturbation:

\[ A(x, 0) = \begin{cases} 
\epsilon & |x - x_c| < \frac{1}{2\rho_0} \\
0 & \text{otherwise.}
\end{cases} \]

- Linear string instability:

\[ \text{Re}(\lambda(k)) > 0 \text{ for some } k, \text{ or} \]

\[ \lim_{t \to \infty} \int dx |A(x, t)| > 0 \]

- Nonlinear instability (metastability):

\[ \lim_{t \to \infty} \int dx |A(x, t)| = 0 \quad \forall \epsilon < \epsilon_{nl} \]

for some \( \epsilon_{nl} > 0 \)

- Convective (meta-)stability:

\[ \lim_{t \to \infty} |A(x, t)| = 0 \]

For any fixed \( x \)
Derivation of the criterion for linear string instability

► General model

\[
\frac{dx_\alpha}{dt} = v_\alpha, \\
\frac{dv_\alpha}{dt} = a_{\text{mic}} \left( s_\alpha(t - T_r), v_\alpha(t - T_r), s_{\alpha-1}(t - T_r), v_{\alpha-1}(t - T_r), \ldots \right)
\]

► Local and instantaneous model

\[
\frac{dv_\alpha}{dt} = a_{\text{mic}} \left( s_\alpha(t), v_\alpha(t), \Delta v_\alpha(t) \right)
\]
Example: The Intelligent-Driver Model (IDM)

- Equations of motion:

\[
\begin{align*}
\dot{x}_\alpha &= v_\alpha, \\
\dot{v}_\alpha &= a \left[ 1 - \left( \frac{v_\alpha}{v_0} \right)^\delta - \left( \frac{s^*(v_\alpha, \Delta v_\alpha)}{s_\alpha} \right)^2 \right]
\end{align*}
\]

- Dynamic desired distance

\[
s^*(v, \Delta v) = s_0 + vT + \frac{v \Delta v}{2 \sqrt{ab}}
\]

Mindest- 
abstand

"Sicherheits"- 
abstand

dynamischer 
Teil
## IDM Model Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Typical value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_0$</td>
<td>120 km/h</td>
</tr>
<tr>
<td>$T$</td>
<td>1.5 s</td>
</tr>
<tr>
<td>$a$</td>
<td>0.3-2.5 m/s²</td>
</tr>
<tr>
<td>$b$</td>
<td>2.0 m/s²</td>
</tr>
<tr>
<td>$s_0$</td>
<td>2 m</td>
</tr>
<tr>
<td>Reaction time $T'$</td>
<td>1-2 s</td>
</tr>
<tr>
<td>Number of observed vehicles</td>
<td>1-4</td>
</tr>
</tbody>
</table>
Example: General macroscopic model

\[
\frac{\partial \rho}{\partial t} + V \frac{\partial \rho}{\partial x} = -\rho \frac{\partial V}{\partial x} + D \frac{\partial^2 \rho}{\partial x^2},
\]

\[
\frac{\partial V}{\partial t} + V \frac{\partial V}{\partial x} = \frac{V_c(\rho) - V}{\tau} - \frac{1}{\rho} \frac{\partial P}{\partial x} + \nu \frac{\partial^2 V}{\partial x^2}
\]

► Stability conditions for both micro and macro models:
Blackboard …
Stability diagram for several models …

(a) Macromodel: GKT model

(b) Car-following model: IDM

(c) Cellular automaton: KKW model

(d) 3-phase car-following model: KK model
One and the same model can adopt several stability classes!

- **Class 1:** Maximum flow unstable
- **Class 2:** Maximum flow (meta-)stable, (convectively) unstable for higher densities
- **Class 3:** Unconditionally stable
- **Subclasses a/b:** No restabilization/restabilization for very high densities

Class 1/2/3:
- \(a=0.3/0.6/2\)
- Class a-b:s1=0/14
3a: Convective instability is really universal!
3c: States for a stability class 2b macroscopic model

GKT model

Class 2b: \(a=0.6 \, \text{m/s}^2, s_1=14 \, \text{m}\)

Class 2a: \(s_1=0\)
3d: Effect of Instationarities at the bottleneck

GKT model

IDM

Flow-conserving bottleneck

Onramp bottleneck: Stationary +Variable part

Onramp bottleneck of same “bottleneck strength”
Again, this mechanism is universal …
Alternative mechanism 2 to GP/pinch effect: Offramp - onramp combinations create this phenomenon as well ...
Alternative explanation 1 for the fundamental diagram: Inter-driver heterogeneity
2D fundamental diagram: Alternative explanation 2: Intra-driver heterogeneity

Variance-Driven Time headways (VDT)

\[ T = T_{\text{free}} \left( 1 + \gamma \frac{\sigma_v}{\langle V \rangle} \right) \]

+2 types
+acceleration fluctuations

![Diagram showing probability density and flow against density and net time headway](image)
2D fundamental diagram: Alternative explanation 3: Dynamical instability

Plain IDM (parameters for stability class 2a)

Upstream of on-ramp bottleneck

At bottleneck
Summary: All three alternative factors for the “2D” nature of the fundamental diagram

- **Deterministic, 2 types, VDT**
- **Stochastic, 1 type, VDT**
Conclusions

- The question whether three or five dynamic phases is essentially one of the definition of a “dynamic phase”.
- There are several mechanisms to explain the observed spatiotemporal features and the 2D fundamental diagrams with “two-phase” models featuring a unique equilibrium relation.
- In many aspects, the discrepancies between Kerner’s approaches and ours are just a result of interpreting things differently.