Frank Jenko

Gyrokinetics – an efficient framework for studying turbulence and reconnection in magnetized plasmas

Max-Planck-Institut für Plasmaphysik, Garching

Workshop on Vlasov-Maxwell Kinetics
WPI, Vienna, 31 March 2011

Many thanks to various co-authors!

Various multiscale challenges in space and astrophysics

Many phenomena involve fluid, ion, electron scales, e.g.:

- magnetic reconnection
- shock waves
- turbulence
- cross-field transport

Simulations are challenged to address these issues.

Current numerical approaches

MHD (including multi-fluid extensions)
…the workhorse…

Kinetic (PIC or Vlasov, including δf versions)
…the microphysics laboratory…

Bridging spatio-temporal scales:
- **large-scale kinetics** (test particle approach)
- **hybrid codes** (e.g. kinetic ions & fluid electrons)

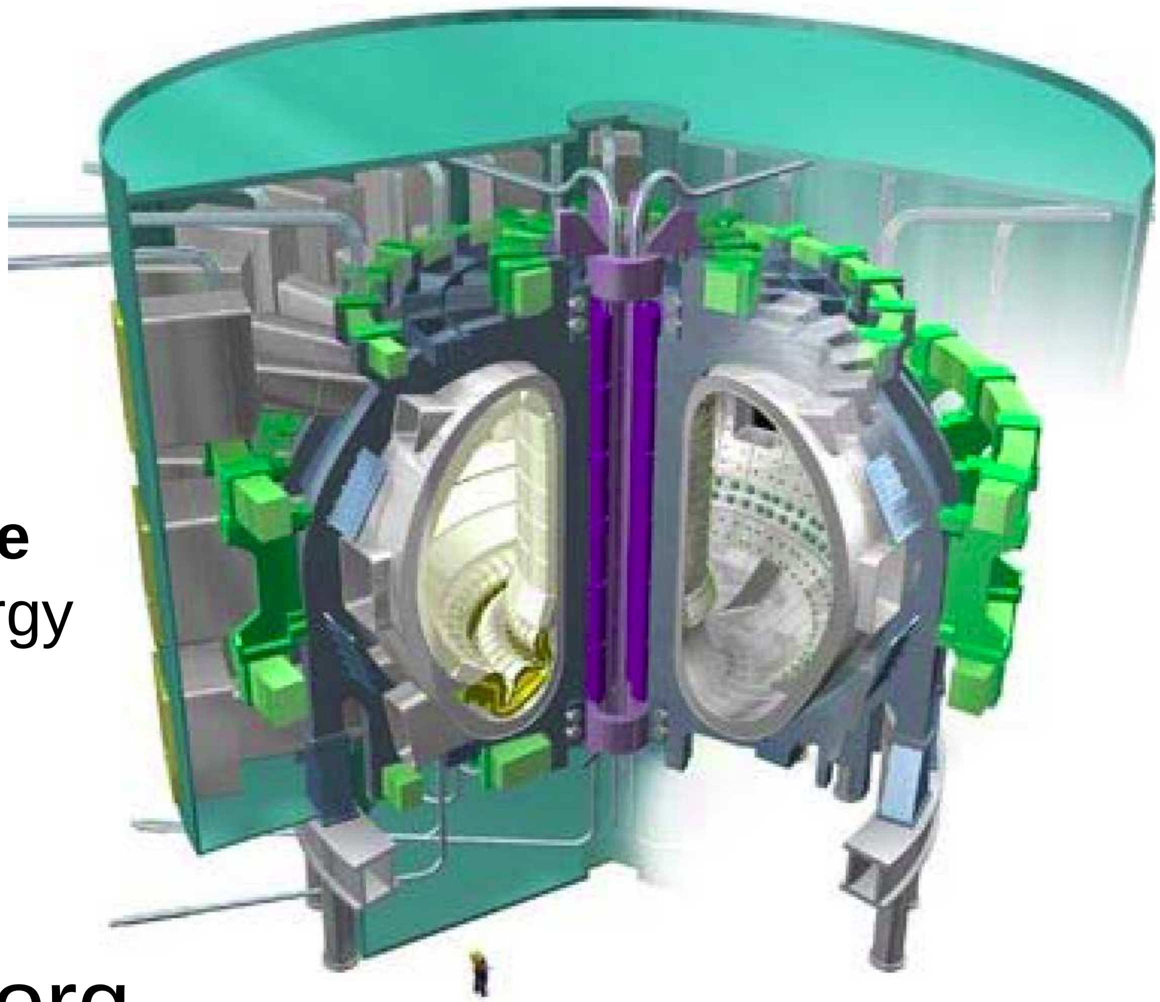
Another option: Gyrokinetics…

From fusion to space plasmas

ITER and plasma turbulence

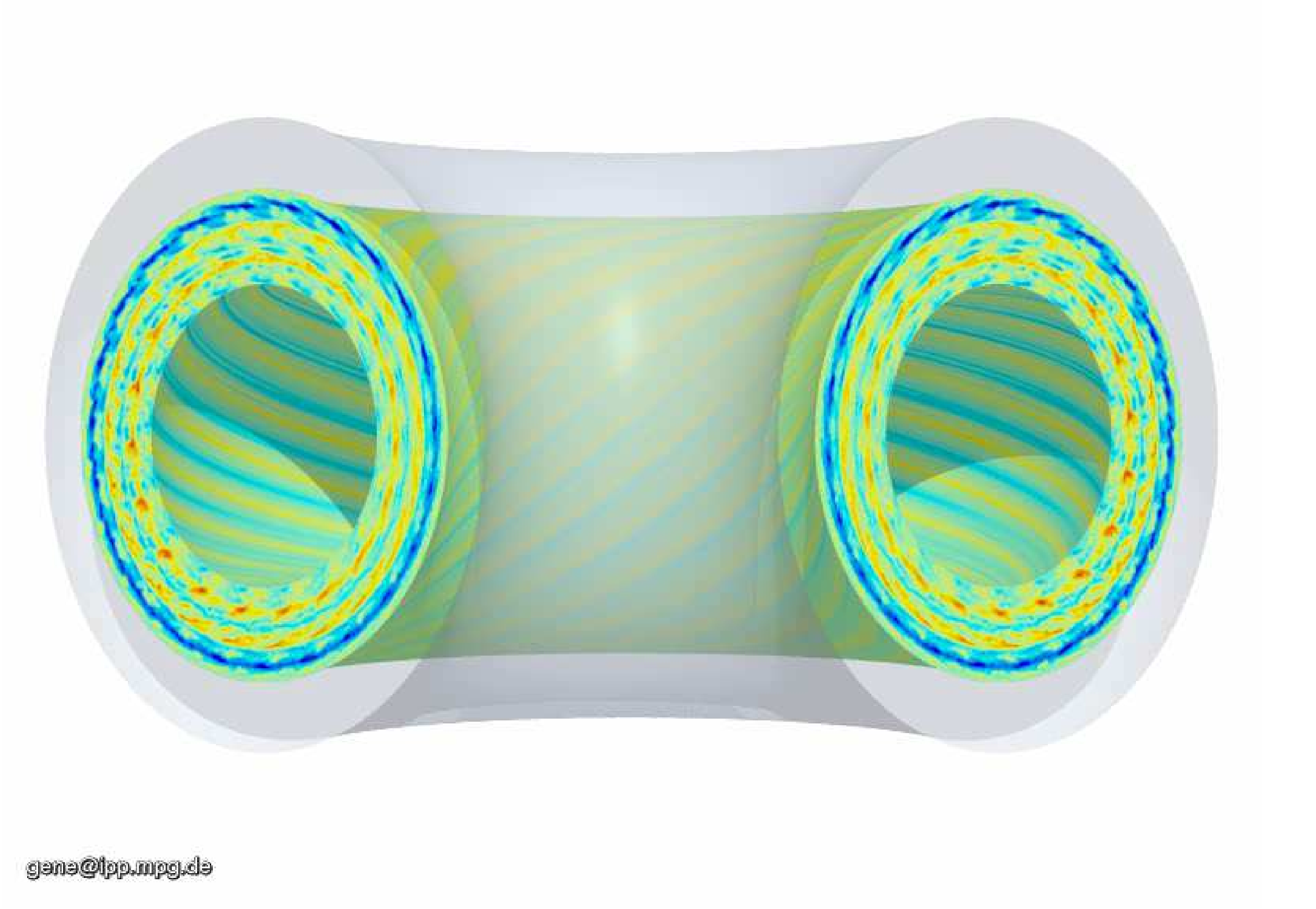
ITER is one of the most challenging scientific projects

Plasma turbulence determines its energy confinement time



www.iter.org

Plasma turbulence: GENE simulations



gene.rzg.mpg.de

Applications of insights, theories, and tools to space plasma physics

Some issues under investigation:

- Solar wind heating
- Magnetic reconnection with guide fields
- Cosmic ray transport
- (…)

Gyrokinetic theory:
A brief guided tour

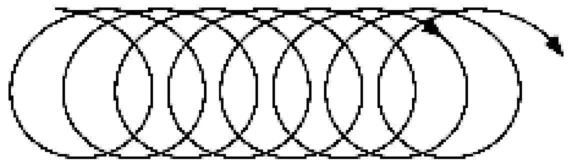
What is gyrokinetic theory?

Dilute and/or hot plasmas are almost collisionless.

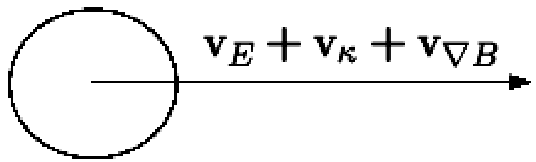
Thus, if kinetic effects (finite Larmor radius, Landau damping, magnetic trapping etc.) play a role, MHD is not applicable, and one has to use a kinetic description!

Vlasov-Maxwell equations $$\left[\frac{\partial}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial \mathbf{x}} + \frac{q}{m}\left(\mathbf{E} + \frac{\mathbf{v}}{c}\times\mathbf{B}\right)\cdot\frac{\partial}{\partial \mathbf{v}}\right] f(\mathbf{x},\mathbf{v},t) = 0$$

Removing the fast gyromotion leads to a dramatic speed-up $\omega \ll \Omega$



Charged rings as quasiparticles; gyrocenter coordinates; keep kinetic effects



Details may be found in: Brizard & Hahm, Rev. Mod. Phys. **79**, 421 (2007)

The gyrokinetic ordering

- The gyrokinetic model is a Vlasov-Maxwell on which the GK ordering is imposed:

⇒ Slow time variation as compared to the gyro-motion time scale:

$$\omega/\Omega_i \sim \epsilon_g \ll 1$$

⇒ Spatial equilibrium scale much larger than the Larmor radius:

$$\rho/L_n \sim \rho/L_T \equiv \epsilon_g \ll 1$$

⇒ Strong anisotropy, i.e. only perpendicular gradients of the fluctuating quantities can be large ($k_\perp \rho \sim 1$, $k_\| \rho \sim \epsilon_g$):

$$k_\|/k_\perp \sim \epsilon_g \ll 1$$

⇒ Small amplitude perturbations, i.e. energy of perturbation much smaller than the thermal energy:

$$e\phi/T_e \sim \epsilon_g \ll 1$$

A. Bottino

A brief historical review

- The word "Gyrokinetic" appeared in the literature in the late sixties.
  Rutherford and Frieman, Taylor and Hastie [1968].

  Goal: Provide a adequate formalism for the linear study of kinetic drift-waves
  in general magnetic configurations, including finite Larmor radius effects.

- First nonlinear set of equations for the perturbed distribution function δF.
  Frieman and Liu Chen [1982].
  $\rightarrow$ Gyrokinetic ordering.

- Littlejohn [1979], Dubin [1983], Hahm[1988], Brizard [1989], ...

  Firm and more transparent theoretical foundation for GK:

  GK equations based on Hamiltonian or Lagrangian variation methods.

Lagrangian
$\Downarrow$
remove gyro-angle dependency in Lagrangian
(change of coordinate system)
$\Downarrow$
equation of motion

A Lagrangian approach

If the Lagrangian of a dynamical system is known...

Example: charged particle motion, in non canonical coordinates $(\vec{x}, \vec{v})$:

$$
\begin{aligned}
L &= \left(\frac{e}{c}\vec{A}(\vec{x},t) + m\vec{v}\right)\cdot\dot{\vec{x}} - H(\vec{x},\vec{v}) \\
H &= \frac{m}{2}v^2 + e\phi(\vec{x},t)
\end{aligned}
$$

with $\vec{B} = \nabla \times \vec{A}$ and $\vec{E} = -\nabla\phi - \partial_t \vec{A}/c$.

...the equation of motion are given by the Lagrange equations:

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = 0 \quad \text{with } i = 1, \ldots, 6$$

Lagrange equation of motion for a charged particle:

$$
\begin{aligned}
\vec{v} \Rightarrow &\quad -\tfrac{\partial L}{\partial \vec{v}} = 0 &&\Rightarrow \dot{\vec{x}} = \vec{v} \\
\vec{x} \Rightarrow &\quad \tfrac{\mathrm{d}}{\mathrm{d}t}\tfrac{\partial L}{\partial \vec{v}} - \tfrac{\partial L}{\partial \vec{v}} = 0 &&\Rightarrow \dot{\vec{v}} = \frac{e}{m}\left(\vec{E} + \frac{\vec{v}}{c}\times\vec{B}\right)
\end{aligned}
$$

Guiding center coordinates

$$L_{DK} = \left(mv_{\parallel}\vec{b} + \frac{e}{c}\vec{A}(\vec{R})\right)\cdot\dot{\vec{R}} + \frac{\mu B}{\Omega}\dot{\varphi} - H_{DK}$$
$$H_{DK} = \frac{m}{2}v_{\parallel}^2 + \mu B + q\phi(\vec{R})$$

- Lagrange equations:

$$\dot{\vec{R}} = v_{\parallel}\vec{b} + \frac{B}{B_{\parallel}^*}\left(\vec{v}_{E\times B} + \vec{v}_{\nabla B} + \vec{v}_C\right)$$
$$\dot{v}_{\parallel} = \left(-\mu\nabla B + e\vec{E}\right)\cdot\frac{\dot{\vec{R}}}{mv_{\parallel}} \quad ; \quad \dot{\mu} = 0 \quad ; \quad \dot{\varphi} = \Omega$$

$\vec{v}_{E\times B} \equiv$	$\frac{c}{B^2}\,\vec{E}\times\vec{B}$	$E\times B$ drift
$\vec{v}_{\nabla B} \equiv$	$\frac{\mu}{m\Omega}\,\vec{b}\times\nabla B$	∇B drift
$\vec{v}_C \equiv$	$\frac{v_{\parallel}^2}{\Omega}\,\vec{b}\times(\vec{b}\cdot\nabla)\vec{b}$	Curvature drift

with $\vec{B}^* \equiv \vec{B} + (mc/e)v_{\parallel}\nabla\times\vec{b} = B(1+\mathcal{O}(\rho_{\parallel}/L_B))$.

Including fluctuating fields

Particle world, $(\vec{x}, \vec{v})$

⇓

static fields

⇓

Guiding center, $(\vec{R}, v_{\parallel}, v_{\perp}, \varphi)$

(remove gyro-angle dependence)

⇓

→ Fluctuating fields

⇓

Gyrocenter coordinates $(\vec{\bar{R}}, \bar{v}_{\parallel}, \bar{v}_{\perp}, \bar{\varphi})$

(remove gyro-angle dependence)

⇓

Equations of motion

Vlasov equation

Self consistent Poisson equation

(Self consistent Ampere's law)

Gyrokinetic Lagrangian 1-form

Eliminate explicit gyrophase dependence via near-identity (Lie) transforms to gyrocenter coordinates:

$$\Gamma \;=\; \left(m v_{||}\mathbf{b}_0 + \frac{e}{c}\,\bar{A}_{1||}\,\mathbf{b}_0 + \frac{e}{c}\,\mathbf{A}_0\right)\cdot d\mathbf{X} + \frac{mc}{e}\,\mu\,d\theta\, - \\ - \left(\frac{m}{2}v_{||}^2 + \mu B_0 + \mu\bar{B}_{1||} + e\,\bar{\phi}_1\right)dt$$

$$\bar{\phi}_1 \equiv I_0(\lambda)\,\phi_1\,, \quad \bar{A}_{1||} \equiv I_0(\lambda)\,A_{1||}\,, \quad \bar{B}_{1||} \equiv I_1(\lambda)\,B_{1||}$$

New Euler-Lagrange equations

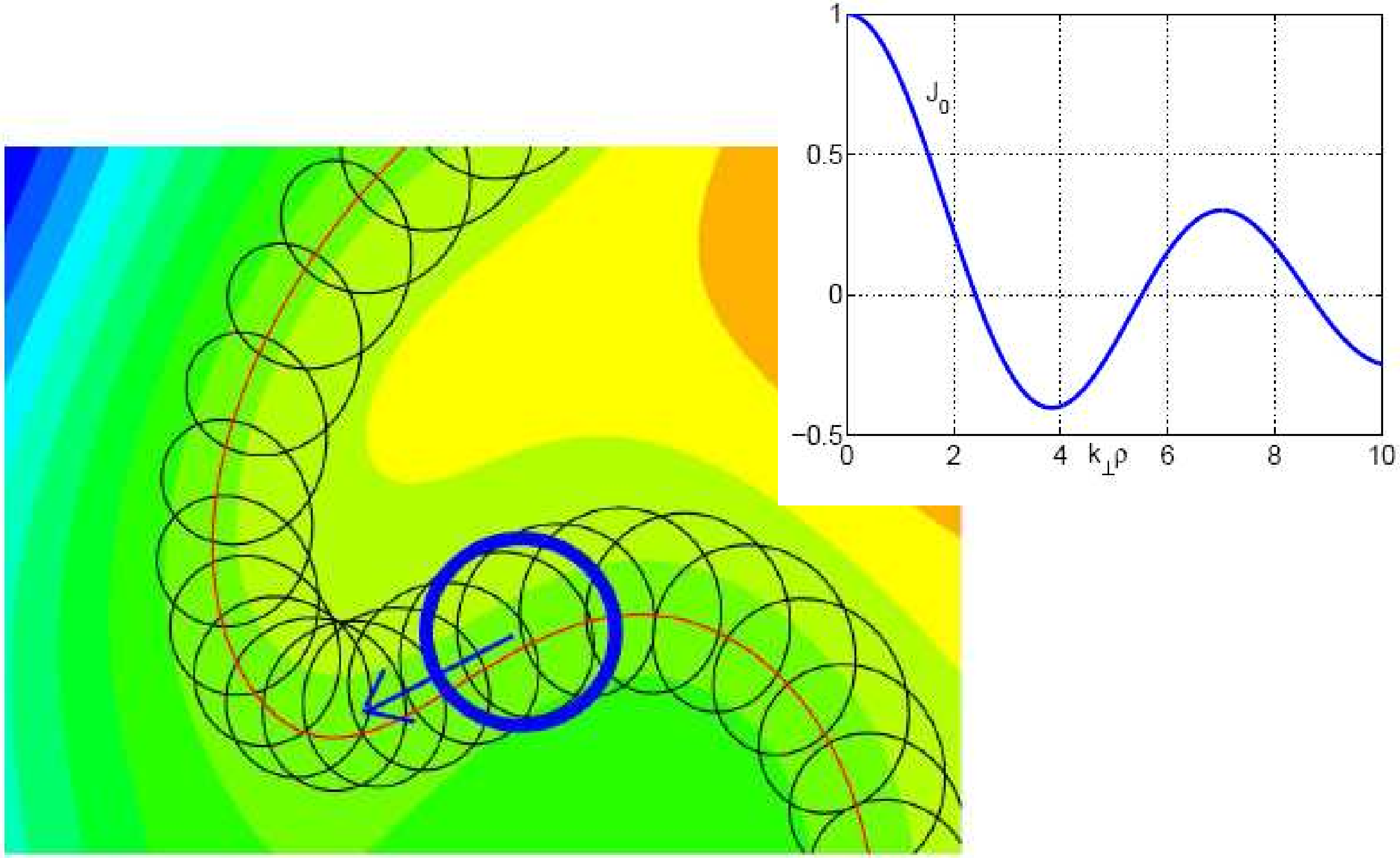
$$\dot{\mathbf{X}} = v_{||}\,\mathbf{b} + \frac{B}{B_{||}^*}\left(\frac{v_{||}}{B}\,\bar{\mathbf{B}}_{1\perp} + \frac{c}{B^2}\,\bar{\mathbf{E}}_1 \times \mathbf{B} + \frac{\mu}{m\Omega}\,\mathbf{b} \times \nabla(B + \bar{B}_{1||}) + \frac{v_{||}^2}{\Omega}\,(\nabla \times \mathbf{b})_\perp\right)$$

$$\dot{v}_{||} = \frac{\dot{\mathbf{X}}}{m v_{||}} \cdot \left(e\bar{\mathbf{E}}_1 - \mu\nabla(B + \bar{B}_{1||})\right) \qquad \dot{\mu} = 0$$

$$f = f(\mathbf{X}, v_{||}, \mu; t)$$

$$\frac{\partial f}{\partial t} + \dot{\mathbf{X}} \cdot \frac{\partial f}{\partial \mathbf{X}} + \dot{v}_{||}\,\frac{\partial f}{\partial v_{||}} = 0$$

Gyroaveraged potentials



- Full Lorentz dynamics
- Gyrokinetic approx.: $\phi^{\text{eff}}(\vec{x},\rho) = \frac{1}{2\pi}\int_0^{2\pi} d\varphi\, \Phi(\vec{x}+\vec{\rho})$

$$= \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty} d\vec{k}\, e^{i\vec{k}\vec{x}}\phi(\vec{k})\, J_0(|\vec{k}|\rho)$$

Appropriate field equations

Reformulate Maxwell's equations in gyrocenter coordinates:

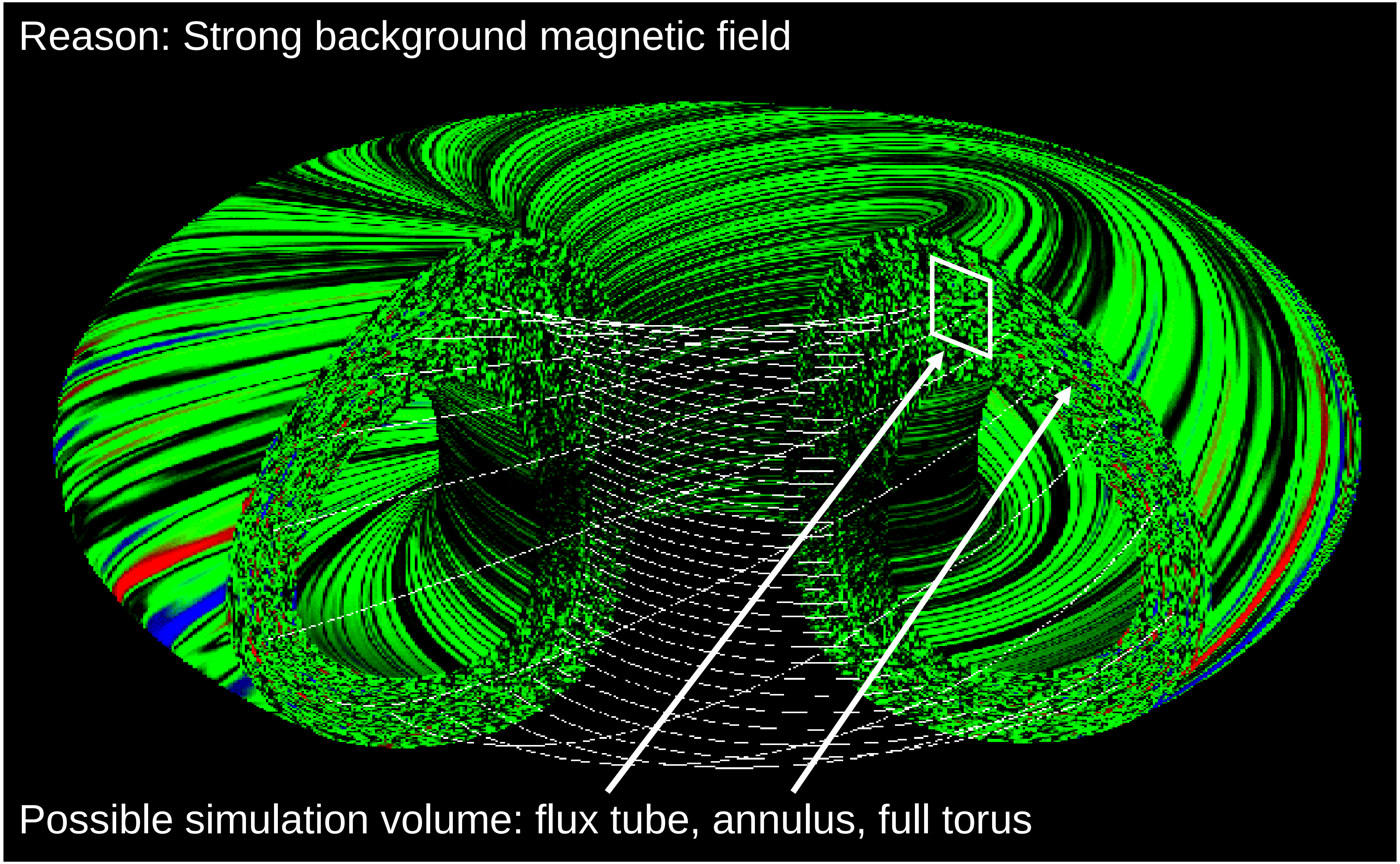
$$\nabla_\perp^2 \phi_1 = -4\pi \sum e n_1 \,, \quad \frac{n_1}{n_0} = \frac{\bar{n}_1}{n_0} - \left(1 - \|I_0^2\|\right) \frac{e\phi_1}{T} + \|x I_0 I_1\| \frac{B_{1\|}}{B} \,,$$

$$\nabla_\perp^2 A_{1\|} = -\frac{4\pi}{c} \sum \bar{J}_{1\|} \,,$$

$$\frac{B_{1\|}}{B} = -\sum \epsilon_\beta \left(\frac{\bar{p}_{1\perp}}{n_0 T} + \|x I_1 I_0\| \frac{e\phi_1}{T} + \|x^2 I_1^2\| \frac{B_{1\|}}{B} \right) \,,$$

Nonlinear integro-differential equations in **5 dimensions**...

Turbulent fluctuations are quasi-2D



Major theoretical speedups

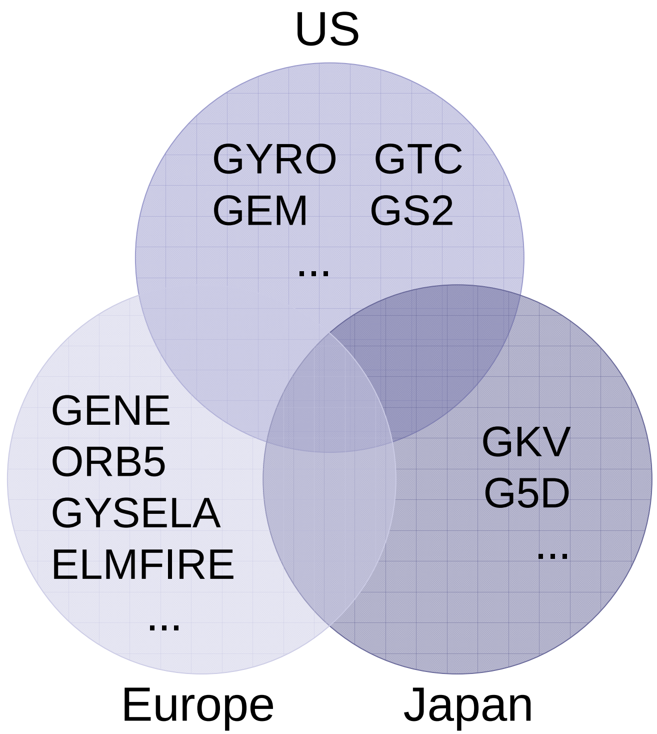
G.Hammett

relative to original Vlasov/pre-Maxwell system on a naïve grid, for ITER $1/\rho_* = a/\rho \sim 1000$

- Nonlinear gyrokinetic equations
  - eliminate plasma frequency: $\omega_{pe}/\Omega_i \sim m_i/m_e$ — x10^3
  - eliminate Debye length scale: $(\rho_i/\lambda_{De})^3 \sim (m_i/m_e)^{3/2}$ — x10^5
  - average over fast ion gyration: $\Omega_i/\omega \sim 1/\rho_*$ — x10^3
- Field-aligned coordinates
  - adapt to elongated structure of turbulent eddies: $\Delta_{||}/\Delta_{\perp} \sim 1/\rho_*$ — x10^3
- Reduced simulation volume
  - reduce toroidal mode numbers (i.e., 1/15 of toroidal direction) — x15
  - $L_r \sim a/6 \sim 160\ \rho \sim 10$ correlation lengths — x6
- **Total speedup — x10^{16}**
- For comparison: Massively parallel computers (1984-2009) — x10^7

Status quo in gyrokinetic simulation

- over the last decade or so, GK has emerged as the standard approach to plasma turbulence
- a variety of nonlinear GK codes is being used and (further) developed
- these codes differ with respect to their numerics, physics, parallel scalability, and public availability



Example: The simulation code GENE

• GENE is physically comprehensive CFD code with applications to both fusion and astrophysical plasmas

• two main goals: deeper understanding of fundamental physics issues and direct comparisons with experiments (interfaces to MHD codes)

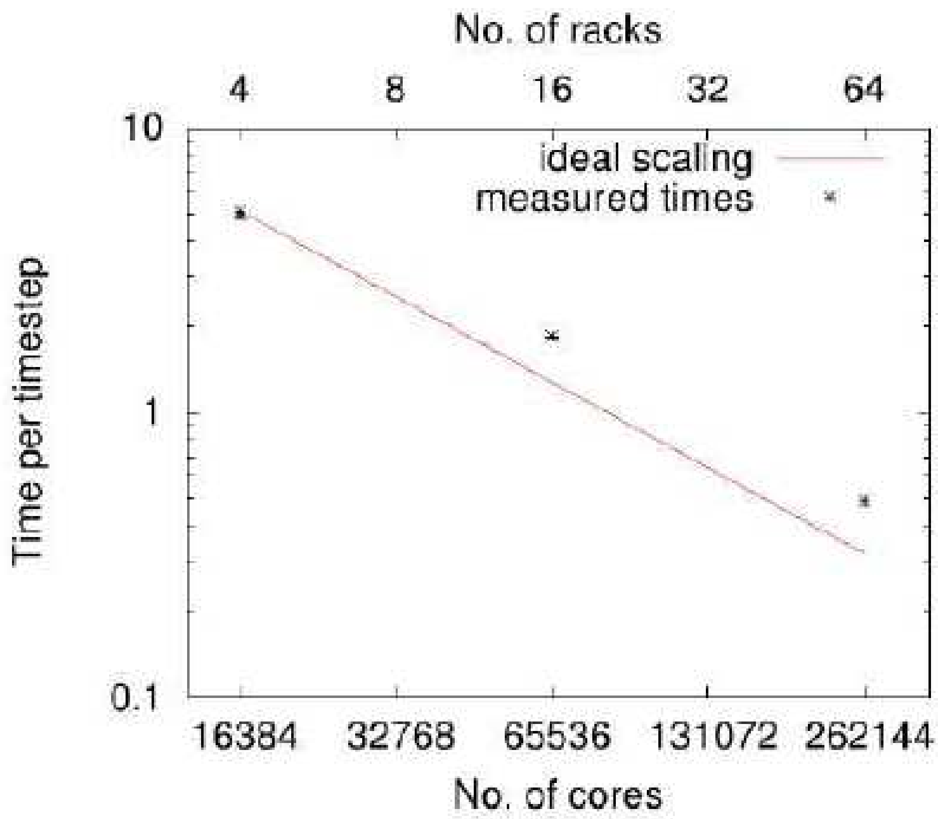
• the differential operators are discretized via a combination of spectral, finite difference, finite element, and finite volume methods; the time stepping is done via a (non-standard) explicit Runge-Kutta method

• GENE is developed cooperatively by an international team, and it is publicly available (gene.rzg.mpg.de)

• GENE is very efficient computationally: parallelization over all phase space coordinates; code automatically adapts to hardware & chosen grid

Gyrokinetic Electromagnetic Numerical Experiment

Massive parallelism

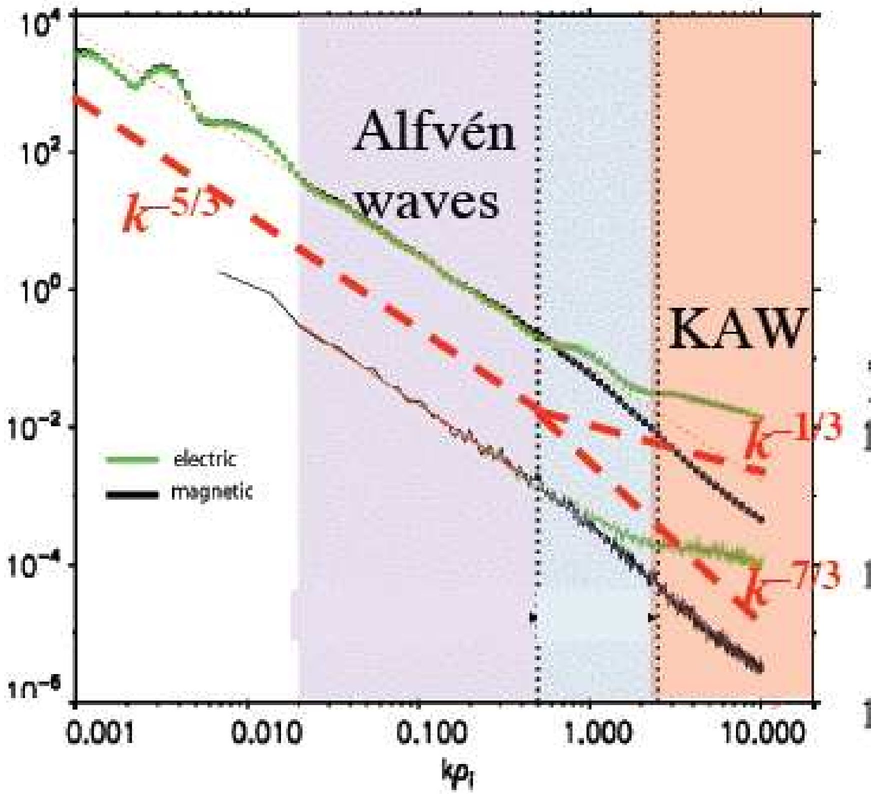


Strong scaling (fixed problem size) of GENE on Jülich's JUGENE system

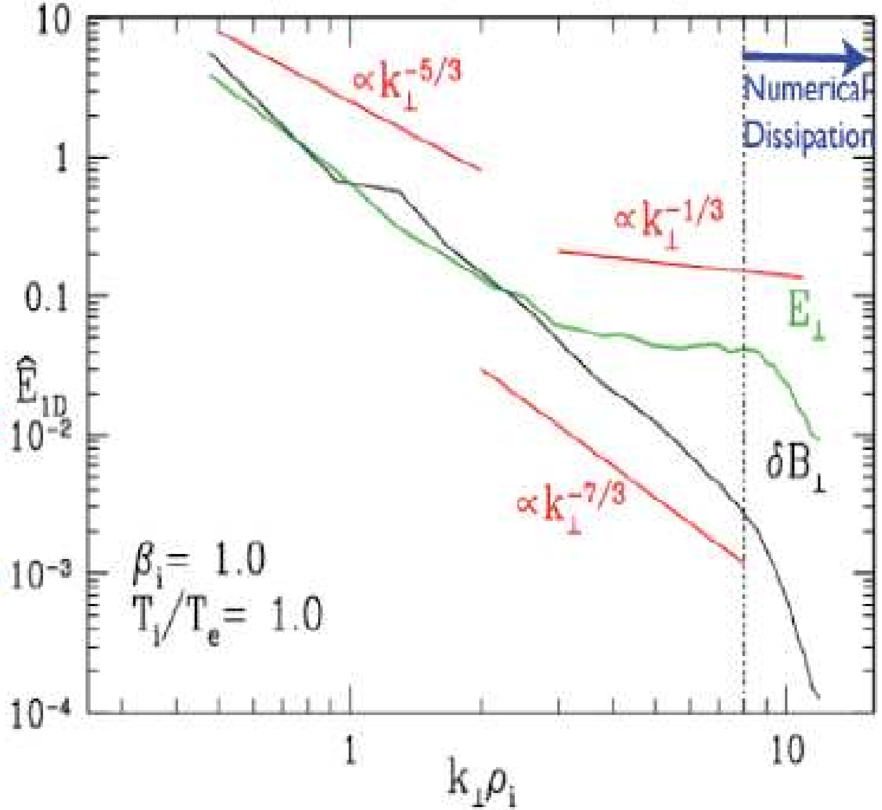
Applications to space and astrophysics

Solar wind turbulence: Dissipation?

AW turbulence in the solar wind
Bale et al., PRL 2005



GK simulation of AW turbulence
Howes et al., PRL 2008



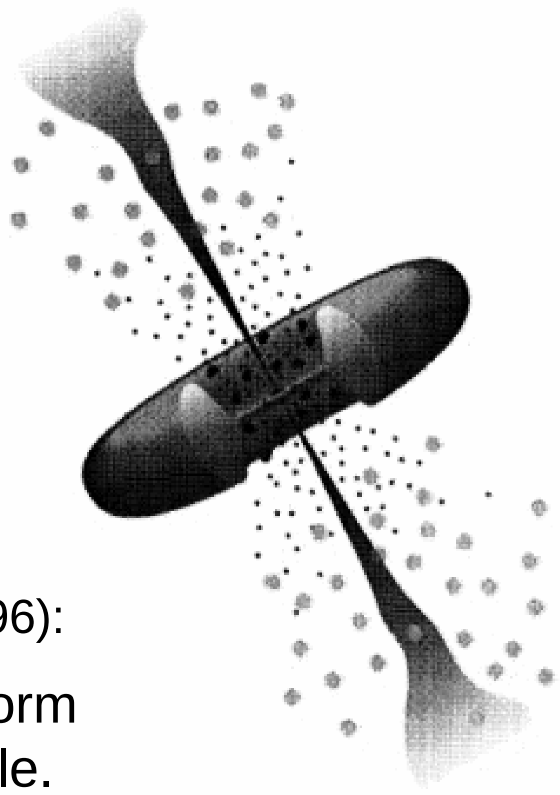
High-k MHD turbulence satisfies the gyrokinetic ordering!

Broad-line regions in AGNs

Measured electromagnetic spectra from AGNs suggest the existence of…

…cold, dense clouds in a hot, dilute, magnetized medium in the central region of AGNs.

How can those cold clouds survive?



Standard model (e.g. Kuncic et al., MNRAS 1996):

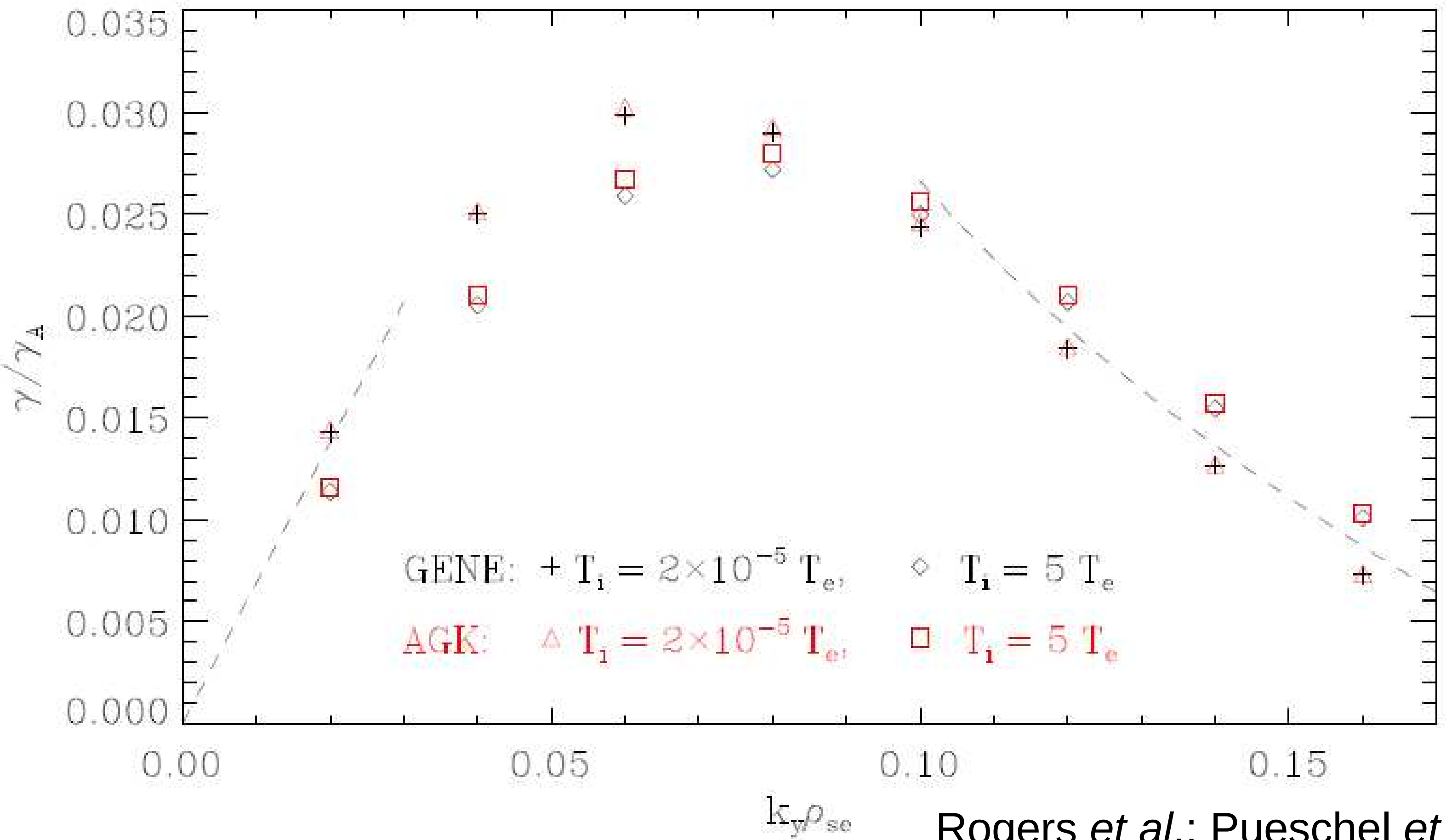
Cold clouds are magnetically confined and form filaments; perpendicular transport is negligible.

Gyrokinetic turbulence sets lower limit on cloud size!

Pueschel *et al.*

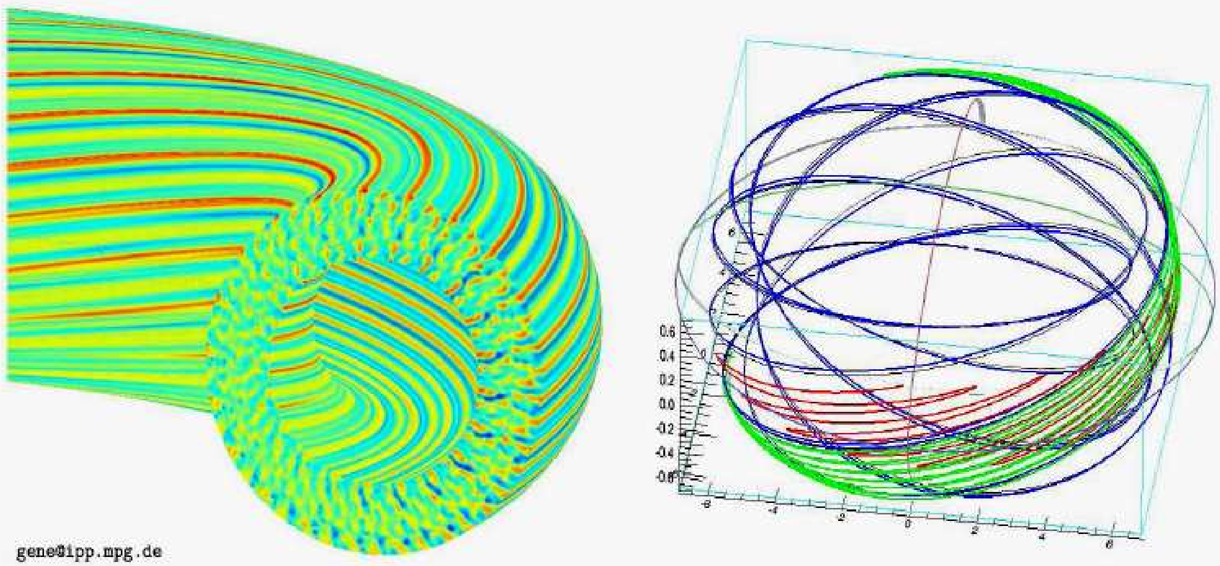
Strong guide-field reconnection

Example: Homogeneous magnetic field plus double current layer



Rogers *et al*.; Pueschel *et al*.

Transport of energetic particles



Turbulence in a torus (GENE)

Drift orbits in the torus

Transport of energetic ions in toroidal plasmas:
T. Hauff *et al*., Physical Review Letters **102**, 075004 (2009)

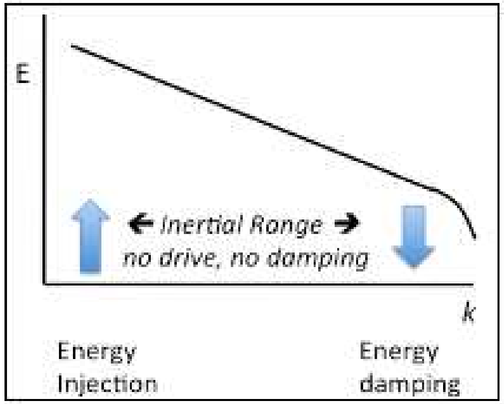
Transport of cosmic rays:
T. Hauff *et al*., Astrophysical Journal **711**, 997 (2010)

Dissipation & cascades in plasma turbulence

Hatch, Terry, Jenko, Merz & Nevins, PRL 2011

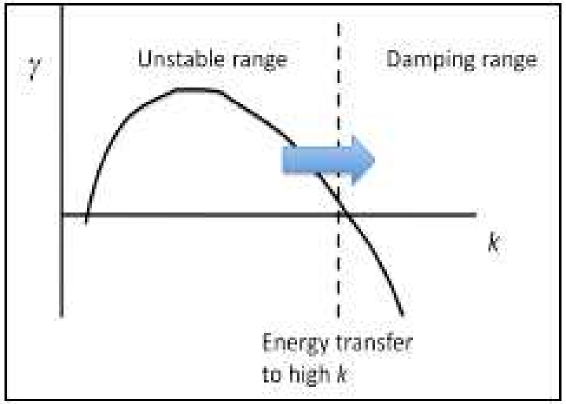
Turbulence in fluids and plasmas – Three basic scenarios

1.Hydrodynamic cascade



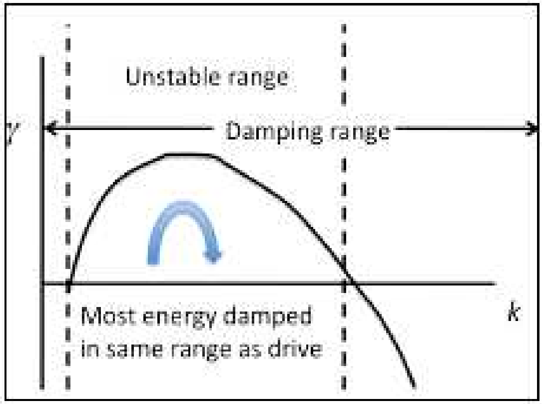
Inertial range
→ no dissipation
→scale invariant dynamics
→power law spectrum

2. Conventional μ-turbulence



Energy transfer to high *k* like hydro – no inertial range adjacent unstable, damping ranges

3. Saturation by damped eigenmode



Energy can go to high *k* but most of it is lost at low *k* in driving range

Saturation via damped eigenmodes

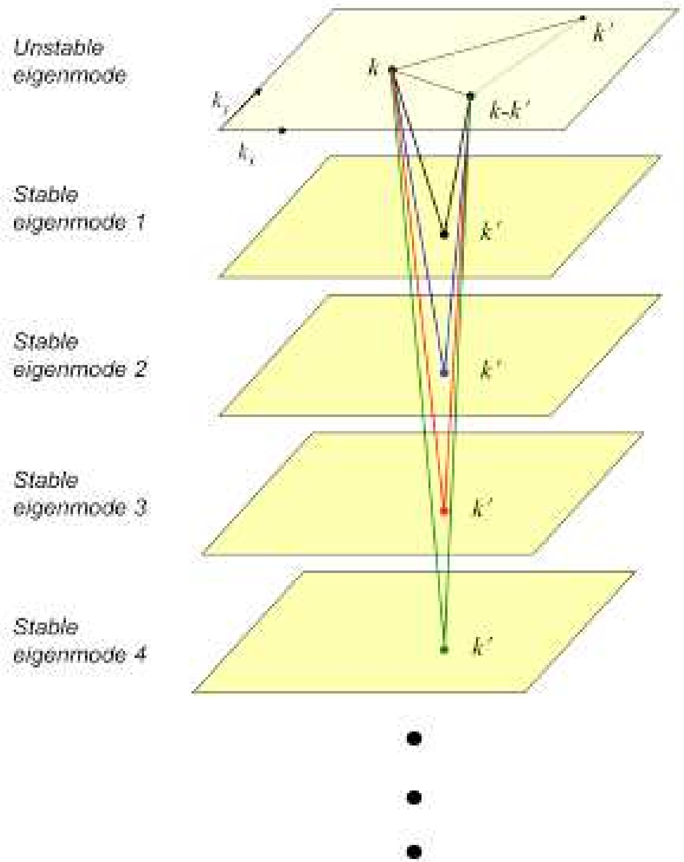
Plasma dispersion relation has multiple roots

- One root unstable → drives turbulence (TEM, ITG, ETG…)
- Other roots can be damped for all *k*
- Fluid models: one root per equation
- Gyrokinetics: infinite in principle;
  discretization yields large but finite number

3-wave interactions drive damped eigenmodes

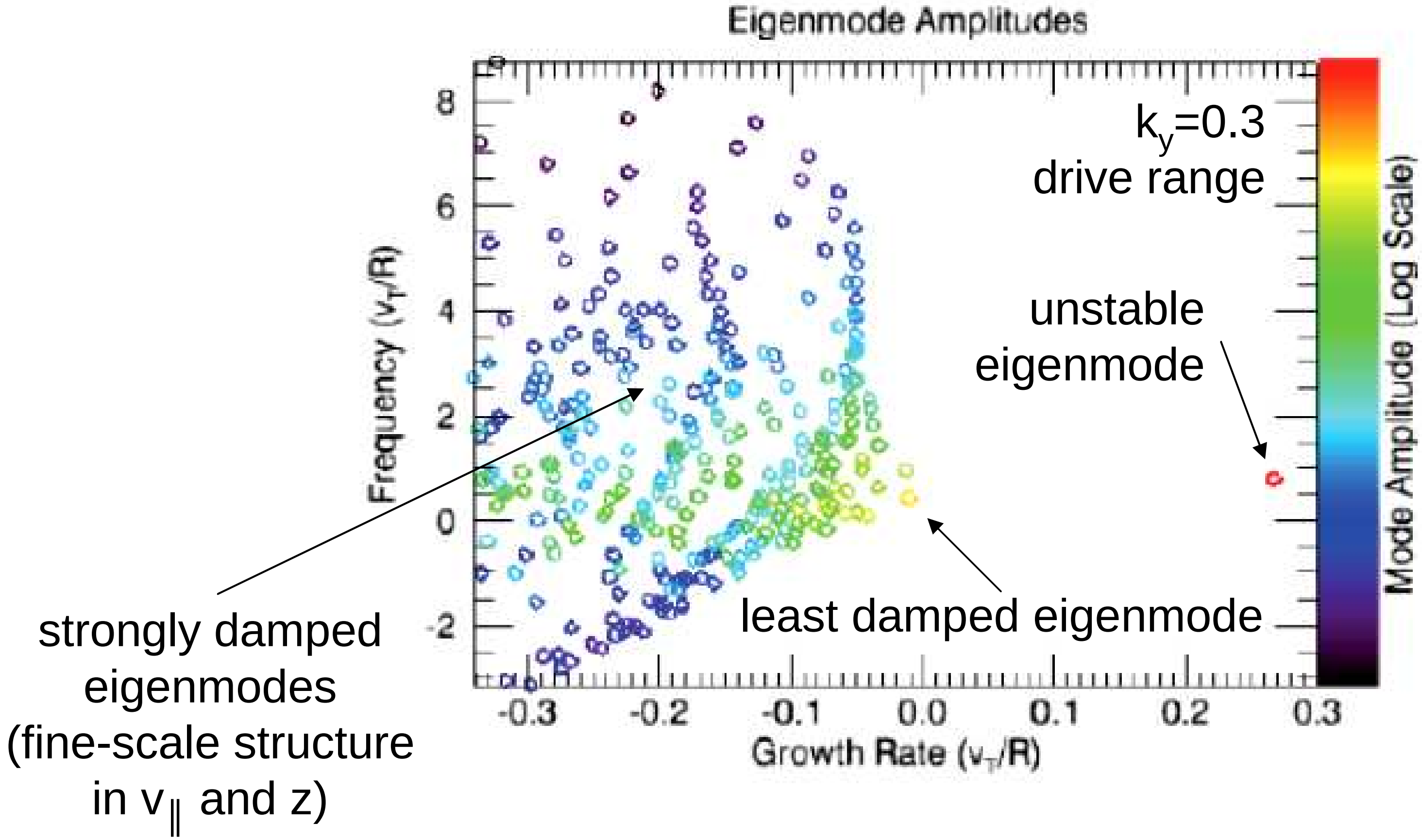
- Pumped by unstable mode through parametric instability
  Only condition: $Amp_{damp} << Amp_{ustable}$ initially
- Each eigenmode driven by combo of all nonlinearities
  => Large *multiplicity* of coupling channels
  => Many eigenmodes are excited

Consistent phenomenology across many models



Excitation of damped eigenmodes

Using GENE as a linear eigenvalue solver to analyze nonlinear ITG runs via projection methods, one finds…



Energetics

Turbulent free energy consists of two parts:

$$\mathcal{E}_f = \sum_j \int d\Lambda \frac{T_{0j}}{F_{0j}} \frac{f_j^2}{2}, \qquad \mathcal{E}_\phi = \sum_j \int d\Lambda q_j \frac{\bar{\phi}_1 f_j}{2}$$

Drive and damping terms:

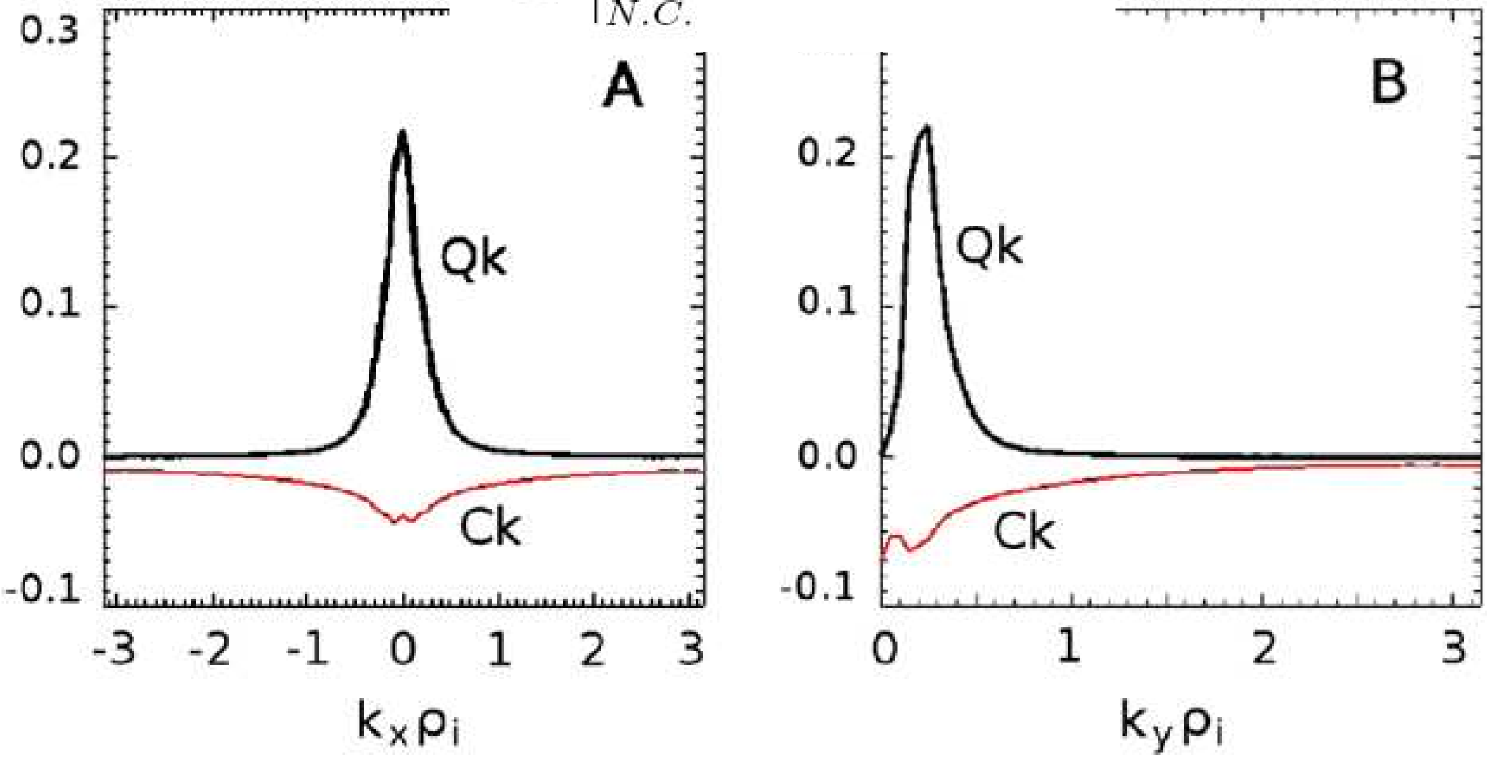
$$\frac{\partial \mathcal{E}}{\partial t} = \sum_j \int d\Lambda \frac{T_{0j}}{F_{0j}} h_j \frac{\partial f_j}{\partial t} = \mathcal{G} - \mathcal{D} \qquad h_j = f_j + (q_j \bar{\phi}_1 / T_{0j}) F_{0j}$$

$$\mathcal{G} = -\sum_j \int d\Lambda \frac{T_{0j}}{F_{0j}} h_j \cdot \left[\omega_n + \left(v_\parallel^2 + \mu B_0 - \frac{3}{2} \right) \omega_{Tj} \right] \times F_{0j} \frac{\partial \bar{\phi}_1}{\partial y}$$

$$\mathcal{D} = -\sum_j \int d\Lambda \frac{T_{0j}}{F_{0j}} h_j (\mathcal{D}_z f_j + \mathcal{D}_{v_\parallel} f_j)$$

Energetics in wavenumber space

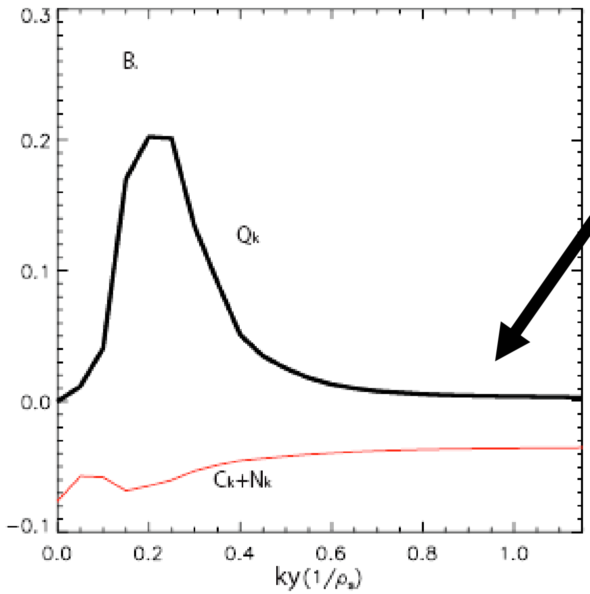
$$\left.\frac{\partial E_k}{\partial t}\right|_{N.C.} = Q_k + C_k + N_k$$



Damped eigenmodes are responsible for significant dissipation in the drive range.

Some energy escapes to high k

From finite amplitude dissipation rate diagnostic, high k dissipation is constant in *k*



Calculate spectrum of residual of energy that is transferred to high *k*

Use attenuation condition:
d/dk (transfer rate) = Energy dissipation rate

Do simple calculation for flow field

Dissipation rate = const. $E(k) = \alpha\, E(k)$

$$E(k) = \int dx\, v^2 e^{ikx}$$

Transfer rate = $T(k) = v_k^3 k$

Use closure of Terry and Tangri, PoP '09

Resulting spectrum decays exponentially @lo k, asymptotes to power law @hi k

Spectrum from k space attenuation of $T(k)$ by dissipation $\alpha E(k)$:

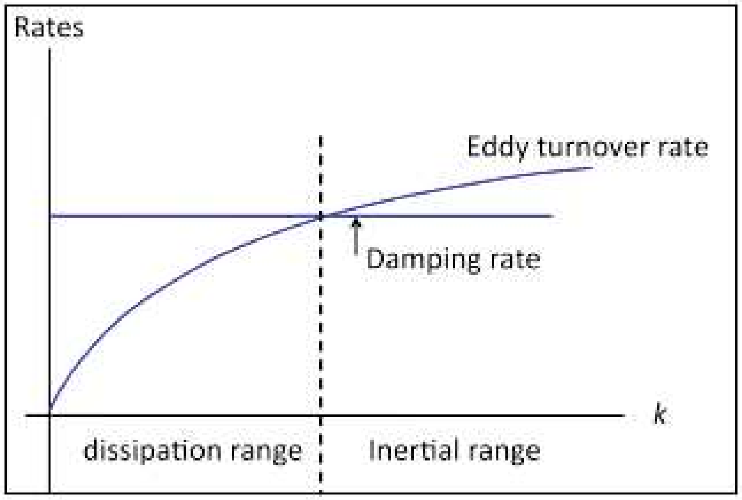
$$\frac{dT(k)}{dk} = \frac{d(v_k^3 k)}{dk} = aE(k)$$

Corrsin closure procedure: $v_k^3 k = v_k^2 \cdot v_k k = E(k)k \cdot \varepsilon^{1/3} k^{-1/3} k$

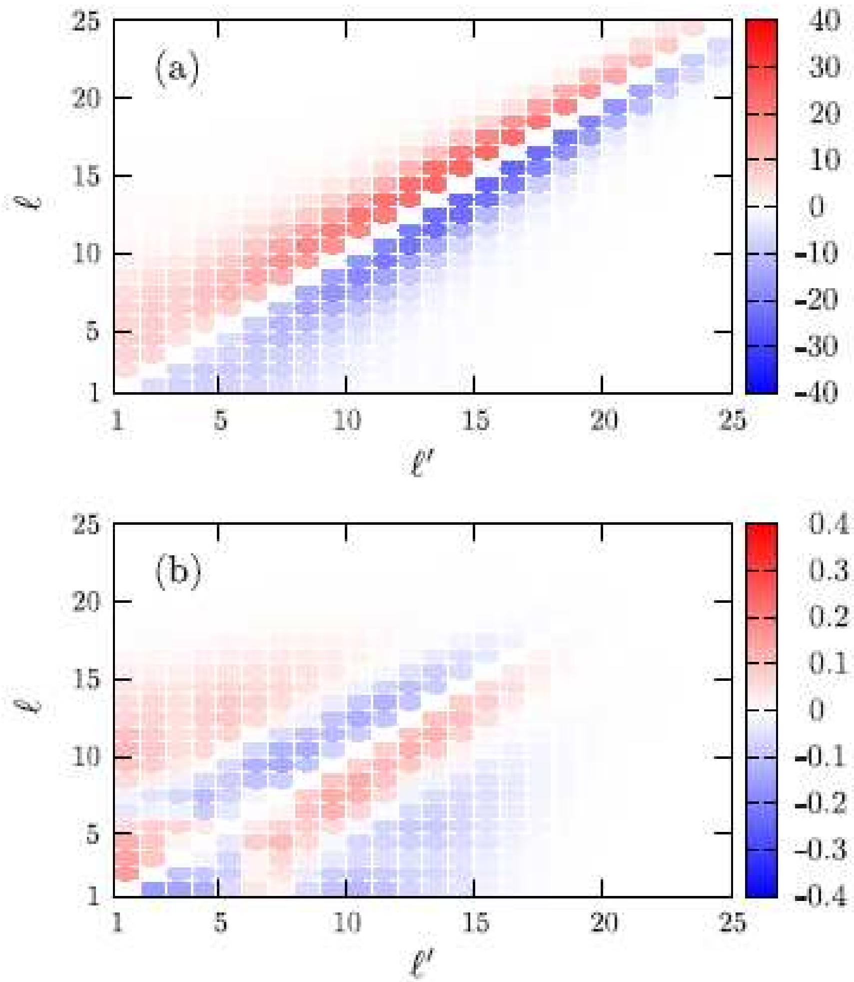
Solving attenuation ODE:

$$E(k) = \beta \varepsilon^{2/3} k^{-5/3} \exp\left[\frac{3}{2}\alpha\varepsilon^{-1/3} k^{-2/3}\right]$$

Spectrum becomes power law in range where eddy turnover rate exceeds constant dissipation rate



Shell-to-shell transfer of free energy



$$\mathcal{E}_f = \sum_i \int d\Lambda \frac{T_{0j}}{F_{0j}} \frac{f_j^2}{2},$$

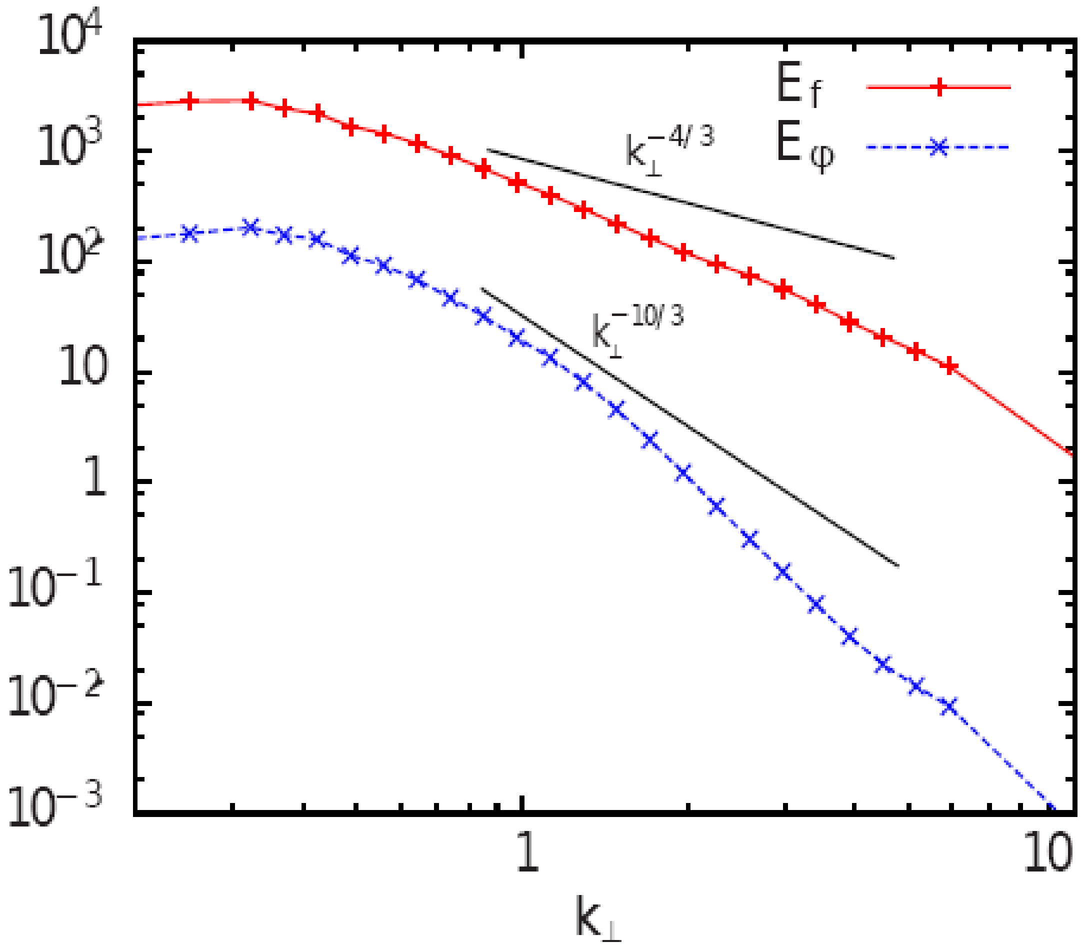
ITG turbulence (adiabatic electrons); logarithmically spaced shells

Entropy contribution dominates; exhibits very local, forward cascade

$$\mathcal{E}_\phi = \sum_j \int d\Lambda q_j \frac{\bar{\phi}_1 f_j}{2}.$$

Banon Navarro *et al*., PRL 2011

Free energy wavenumber spectra



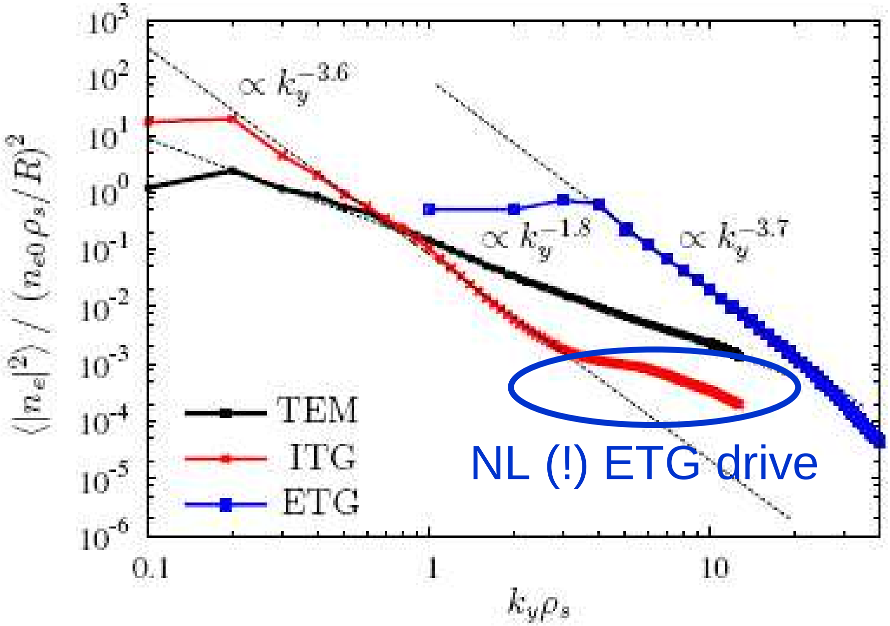
Asymptotic self-similarity coincides with power law spectra

Measured exponents are relatively close to those of a 2D GK scaling theory [Schekochihin *et al*., 2009]

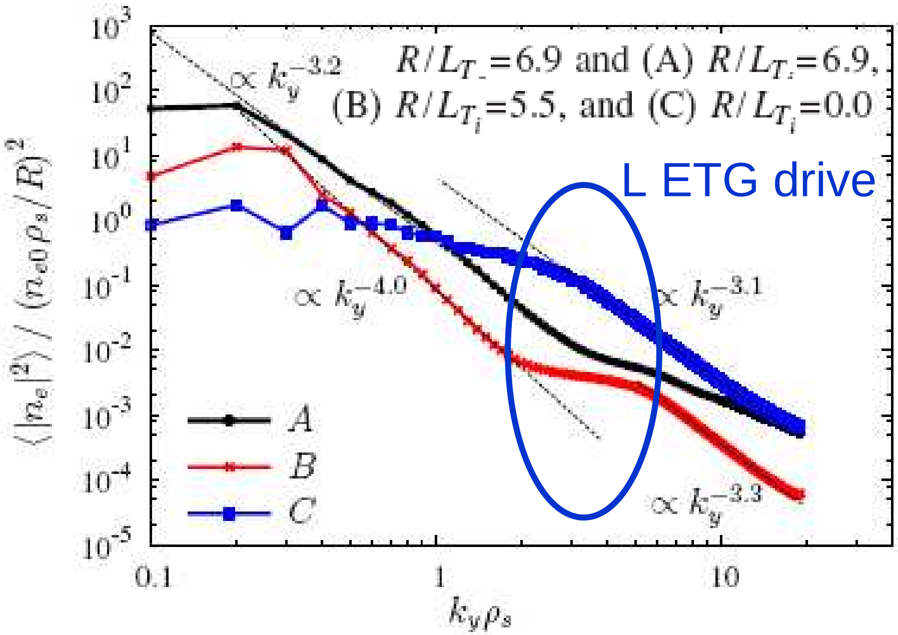
Banon Navarro *et al*., PRL 2011

Multiscale wavenumber spectra

Poloidal wavenumber spectra of density fluctuations for pure TEM / ITG / ETG turbulence



Poloidal wavenumber spectra of density fluctuations for mixed TEM / ITG – ETG turbulence



Universality?

Görler & Jenko, PoP 2008

Application: Gyrokinetic LES models

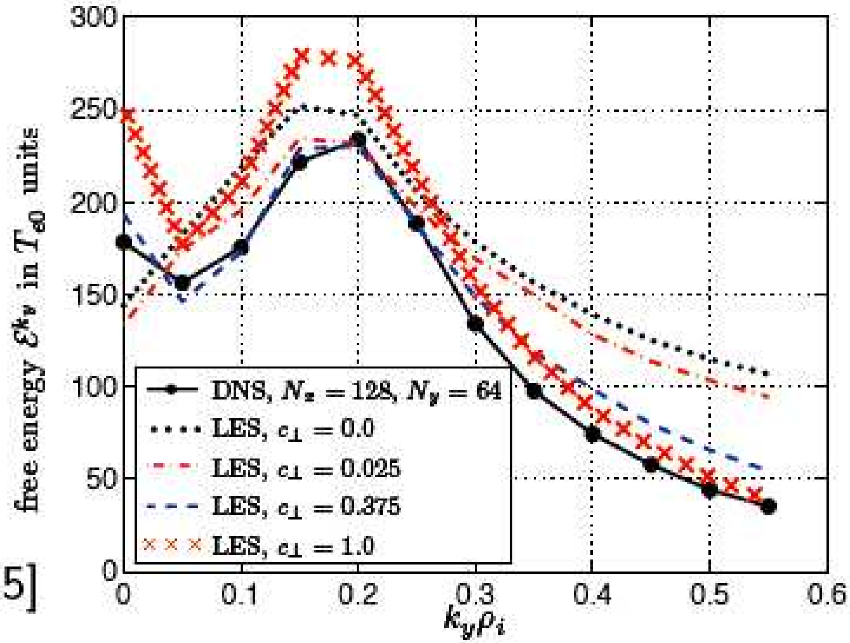
Model: $M[c_\perp, \overline{f}] = -c_\perp k_\perp^4 \overline{f}$

Unknown free parameter: $c_\perp$

Free energy spectra vs $c_\perp$:

Cyclone Base Case (ITG)

- ⋆ $c_\perp$ too small
  - ⇒ not enough dissipation
- ⋆ $c_\perp$ too strong
  - ⇒ overestimates injection
- ⋆ $c_\perp = 0.375$ good agreement
  - → "plateau" for $c_\perp \in [0.25, 0.625]$
  - → holds for k_x



Morel *et al.*, submitted

Outro

GK – A “new” multiscale approach

Whenever a magnetized plasma satisfies the gyrokinetic ordering, one should seriously consider using **gyrokinetics**.

Interesting applications (just a few examples):

- Cascade physics; heating of the solar wind
- Fast reconnection with strong guide fields
- Cross-field transport (e.g. of cosmic rays)

More info:

www.ipp.mpg.de/~fsj

gene.rzg.mpg.de