Efficient mesh adaptation methods for evolution problems: theory and applications

Organization: WPI, Vienna

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14 - 17 December 2011

Lecture program

	Wednesday 14.12	Thursday 15.12	Friday 16.12	Saturday 17.12
09:30 - 10:15	**Ch. Makridakis** “Self adapted methods for evolution PDEs”	**D. Kröner** “Dynamically adapted unstructured grids, parallelization and load balancing for flow simulation”	**M. Feistauer** “Numerical simulation of compressible flow in time-dependent domains and FSI”	**Free discussion**
10:15 - 11:00	**A. Kurganov** “New Adaptive Artificial Viscosity Method for Hyperbolic Systems of Conservation Laws”	**P. G. LeFloch** “Undercompressible shocks and moving phase boundaries”	**A. Chertock** “A simple eulerian finite volumes methods for compressible fluids in domains with moving boundaries”	**Free discussion**
11:00 - 11:30	Coffee break	Coffee break	Coffee break	
11:30 - 12:15	**N. Sfakianakis** “Total variation diminishing and entropy dissipation properties of AMR”	**Ch. Steiner & S. Noelle** “Efficient timestep adaptation for unsteady compressible flows via adjoint error control” & “A conservative formulation of the adjoint problem in adjoint error control of hyperbolic systems”	**G. Rusnakova** “Kinematical splitting of flow in moving domains”	
12:15 - 13:45	Lunch break	Lunch break	Lunch break	
13:45 - 14:30	**Free discussion**	**Free discussion**	**Free discussion**	
14:30 - 15:15	**G. Puppo** “Entropy residuals as a posteriori error indicators”	**K. Mikula** “Numerical modelling of forest fire propagation”	**D. Praetorius** “Convergence and quasi-optimality of adaptive boundary element methods ”	
15:15 - 16:00	**Th. Katsaounis** “Adaptive mesh refinement strategies for two applications: the linear Schrodinger equation and Shear Band formation”	**P. Frolkovic** “Flux-based level set methods for some evolution problems using mesh adaption”	**M. Semplice** “Grid adaptivity for systems of conservation laws”	
16:00 - 16:15	Coffee break	Coffee break	Coffee break	
16:15 - 17:00	**Ch. Simeoni** “Remarks on the consistency of upwind source at interface schemes on nonuniform grids”	**Free discussion**	**Free discussion**	

Organization comments

Accommodation

Accommodation costs are covered by WPI.

Upon arrival to WPI

WPI is on the 7th floor, corridor C of the mathematics department (directions from the hotels have been sent from Mrs. Obermaier).
There you can find the lecture room (C 714), an office with your name on, and the office of Mrs. Preuss and Mrs. Obermaier; please check in with them first.
More information like internet access will be provided on site.

Coffee breaks

Coffee and cakes are provided in the lecture room. They are offered by WPI.

Lunch breaks

Either in local MENSA or in neighboring restaurants. Cost is covered by the participants.

Social evening

Place: Zum Martin Sepp (http://www.zummartinsepp.at/) in Grinzig.
Date: Thursday 15.12
Time: 17:30 - 18:00 (public transportation, departure from WPI)
Drinks: Offered by WPI.
Food: Cost is covered by the participants.

A. Kurganov: "New Adaptive Artificial Viscosity Method for Hyperbolic Systems of Conservation Laws"

We propose a new finite volume method for solving general multidimensional hyperbolic systems of conservation laws. Our method is based on an appropriate numerical flux and a high-order piecewise polynomial reconstruction. The latter is utilized without any computationally expensive nonlinear limiters, which are typically needed to guarantee nonlinear stability of the scheme. Instead, we enforce stability of the proposed method by adding a new adaptive artifcial viscosity, whose coefficients are proportional to the size of the weak local residual, which is sufficiently large at the shock regions, much smaller near the contact waves, and very small in the smooth parts of the computed solution. We test the proposed scheme on a number of benchmarks for both scalar conservation laws and for one- and two-dimensional Euler equations of gas dynamics. The obtained numerical results clearly demonstrate the robustness and high accuracy of the new method.

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N. Sfakianakis: "Total variation diminishing and entropy dissipation properties of AMR"

In this lecture we focus on the mesh adaptation and it stabilization properties. We first present a numerical study of several oscillatory numerical schemes and their behavior under a specific mesh adaptation technique. We then examine the stabilization properties of adaptive mesh refinement techniques by investigating the total variation and entropy dissipation of these schemes. In this talk are presented results of join work with Ch. Arvanitis, Ch. Makridakis, P.G.LeFloch and M. Lukacova.

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G. Puppo: "Entropy residuals as a posteriori error indicators"

Systems of conservation laws deriving from physical models are endowed with entropy inequalities which select unique weak solutions. Entropy is dissipated on shocks giving irreversible solutions. On the contrary, no entropy dissipation occurs on smooth solutions. However, a numerical scheme induces a discretization of the conservation laws on which entropy dissipation occurs even on smooth solutions. This is the entropy residual we will use as a posteriori error indicator. This quantity has the same size of the local truncation error on smooth solutions, while it is a Dirac mass, proportional to the schock strength, on discontinuous waves. In this talk we will construct the entropy residual, and illustrate its main properties. We will also include a discussion of entropy residuals for entropy stable schemes. In this case, the indicator can be obtained as a byproduct of the advancement of the solution in time, and therefore it is a particularly cheap error indicator.

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Th. Katsaounis: “Adaptive mesh refinement strategies for two applications: the linear Schrodinger equation and Shear Band formation”

In this talk I will present two different mesh refinement strategies for two different applications. For the case of the linear Schrodinger equations new aposteriori error estimates, in space and time will be presented. Numerical results of an adaptive algorithm based on these estimates will be presented. Shear Band formation in materials and localization in plasticity is the second application. In these type of applications the solution is highly localized and may blow up under certain conditions. For accurate numerical simulations and adaptive strategy is mandatory to follow the solution up to blow up . In this case the mesh refinement is based on inverse inequalities that finite element spaces satisfy.

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Ch. Simeoni: "Remarks on the consistency of Upwind Source at Interface schemes on nonuniform grids"

The recent years have seen a significant development in the use of nonuniform grids for the numerical solution of partial differential equations. In fact, the use of Cartesian grids composed of rectangular cells does not allow for optimal representation of complex geometries (for example, to handle the presence of cut cells at the boundaries of the computational domain) and, therefore, numerical methods employing either boundary fitted coordinate systems or unstructured grids (finite volumes, finite elements, ...) are a well established way to overcome this problems. This development has given rise to a number of new problems regarding the analysis of such methods: firstly, on nonuniform grids, many formally inconsistent schemes converge; on the other hand, some consistent methods diverge on nonuniform grids in spite of reducing, when used on uniform grids, to standard convergent schemes. We shall report on a numerical study of the properties of supra-convergence for hyperbolic conservation laws with geometrical source terms, which has confirmed that the standard consistency conditions for numerical fluxes do not guarantee that the (local) truncation error vanishes in presence of nonuniform meshes. Nevertheless, the main issue of an error analysis with optimal rates can be pursued, by virtue of the results obtained on the supra-convergence phenomenon for numerical approximation of hyperbolic conservation laws. More clearly, despite the fact that a deterioration of the point-wise consistency is observed in consequence of the non-uniformity of the mesh, the formal accuracy of the methods is actually maintained as the global error behaves better than the truncation error would indicate. This property of enhancement of the numerical error has been widely explored for homogeneous problems, and we attempt at extending such theory to conservation laws with geometrical source terms that are discretized by means of well-balanced schemes, as suggested by the classical application to the Saint-Venant equations for shallow waters. It is worth remarking that the results announced above cannot affect the case of ordinary differential equation with parameter-dependent (geometrical) source terms, namely for systems with negligible fluxes. In effects, elementary counter-examples show that (strong) convergence fails for nonuniform grids, and then some specific approach has to be designed for recovering the error analysis for finite volume schemes on nonuniform meshes. Because we are interested in pointing out numerical issues, we shall present an experimental error analysis to elucidate the influence of the non-uniformity of the mesh mainly on the convergence's rates. Precise comments on the limits and potentiality of these approaches will be done.

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D. Kröner: “Dynamically adapted unstructured grids, parallelization and load balancing for flow simulation”

In this contribution we will report on numerical experiments concerning flow simulation on unstructured dynamically adapted grids in 2 and 3 space dimensions. Since the computation, based on dynamically, locally adapted grids, has been performed on parallel computers it is also necessary to implement an efficient load balancing. The simulations are based on the discretization of the compressible Euler and Navier Stokes equations. The design of grid indicators is still an open and difficult problem. For model problems, this means nonlinear scalar conservation laws in multi dimensions , we were able to prove some rigorous a posteriori error estimators. In this contribution I will report on some joint results together with R. Klöfkorn, Mario Ohlberger and T. Gessner.

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P. G. LeFloch: "Undercompressible shocks and moving phase boundaries"

This lecture will present a domain of applications in which the efficient mesh adaptation techniques presented in this workshop should provide a particularly useful tool. Regularization-sensitive wave patterns often arise in continuum physics, especially in complex fluid flows, which may contain undercompressive shock waves and moving phase boundaries. I will review here the theory of solutions to nonlinear hyperbolic systems of conservation laws, in the regime when small-scale effects like viscosity and capillarity drive the selection and dynamics of (nonclassical) shocks. The concept of a kinetic relation was introduced and provides the proper tool in order to characterize admissible shocks. The kinetic relation depends on higher-order terms that take additional physics into account. A general theory of the kinetic relation has been developed by the author and his collaborators, which covers various issues such as the Riemann problem, the Cauchy problem, the front tracking schemes, and several numerical strategies (schemes with controled dissipation) adapted to handle nonclassical shocks. Relevant papers are available at the link: philippelefloch.org.

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Ch. Steiner: "Efficient timestep adaptation for unsteady compressible flows via adjoint error control" & S. Noelle: "A conservative formulation of the adjoint problem in adjoint error control of hyperbolic systems"

We report on adaptive timestep control for weakly instationary problems. The core of the method is a space-time splitting of adjoint error representations for target functionals due to Süli and Hartmann. It provides an efficient choice of timesteps for implicit computations of weakly instationary flows. The main new ingredients are
(i) a conservative formulation of the dual problem
(ii) the derivation of boundary conditions for a new formulation of the adjoint problem
(iii) the coupling of the adaptive time-stepping with spatial adaptation.
Due to Galerkin orthogonality, the dual solution does not enter the error representation as such. Instead, the relevant term is the difference of the dual solution and its projection to the finite element space. We can show that it is therefore sufficient to compute the spatial gradient of the dual solution. This gradient satisfies a conservation law instead of a transport equation, and it can therefore be computed with the same algorithm as the forward problem, and in the same finite element space. For this new conservative approach we will derive boundary conditions.
For the spatial adaptation, we use a multiscale-based strategy developed by S. Müller (IGPM, RWTH Aachen), and we combine this with an implicit time discretization. The combined space-time adaptive method provides an efficient choice of timesteps for implicit computations of weakly instationary flows. The timestep will be very large in regions of stationary flow, and becomes small when a perturbation enters the flow field. First we demonstrate the capabilities of the approach for a weakly instationary test problem for scalar, 1D conservation laws. Then we extend the computations to the 2D Euler equations, where we couple the adaptive time-stepping with spatial adaptation. The combined space-time adaptive method provides an efficient choice of timesteps for implicit computations of weakly instationary flows. The timestep will be very large in regions of stationary flow, and becomes small when a perturbation enters the flow field. The efficiency of the solver is investigated by means of an unsteady inviscid 2D flow over a bump.

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K. Mikula: "Numerical modelling of forest fire propagation"

We present a mathematical model for numerical modelling of a wind-driven forest fire front propagation. The model is based on evolution of plane curve (representing the fire front) in the outer normal direction by a speed given by the properties of a fuel bed and a wind speed projected onto the normal to the front. The influence of the front shape is modelled by adding the curvature regularization to the normal velocity. For the numerical modelling we use so-called Lagrangean approach where the crucial point is an asymptotically uniform tangential redistribution of grid points which prevents the moving front from forming spurious crossovers and swalow tails. Thanks to the asymptotically uniform tangential grid point redistribution we can also solve detection of topological changes in moving front in O(n) complexity which makes our Lagrangean approach highly efficient and represent significant computational improvement of the existing numerical models used for the forest fire propagation and, in general, it represent new fast and stable method for solving free boundary problems modelled by moving fronts with possible topological changes. This is a common work with Martin Balazovjech and Jozef Urban.

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P. Frolkovic: "Flux-based level set methods for some evolution problems using mesh adaption"

In this talk the flux-based level set methods for modelling of moving interfaces and boundaries for some evolution problems will be discussed. Recent application for problems of groundwater flow with moving water table will be introduced in details. The main ingredients of level set methods like computations of signed distance function, extrapolation of missing data and solution of advection equation will be described. The role of mesh adaption methods will be discussed.

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M. Feistauer: "Numerical simulation of compressible flow in timedependent domains and FSI"

The lecture will be concerned with the simulation of viscous compressible flow in time dependent domains. The motion of the boundary of the domain occupied by the fluid and the solution of the problem on moving meshes is taken into account with the aid of the ALE (Arbitrary Lagrangian-Eulerian) formulation of the compressible Navier-Stokes equations. This system is coupled with equations describing the behaviour of elastic structures under the action of a moving gas. We consider compressible flow in a channel. The part of its walls is formed by an elastic body whose deformation is described by the dynamical elasticity equations. This model is used for the simulation of airflow in human vocal folds. Compressible flow is discretized by the discontinuous Galerkin finite element method (DGFEM) using piecewise polynomial discontinuous approximations. The time discretization is based on a semi-implicit linearized scheme, which leads to the solution of a linear algebraic system on each time level. The developed technique appears unconditionally stable and robust with respect to the magnitude of Reynolds and Mach numbers. It allows the solution of flows with very low Mach numbers as well as high-speed flow. The solution of dynamical elasticity equations is realized with the aid of conforming finite elements. The fluid-structure interaction is realized via the weak and strong coupling. Some results of numerical tests will be presented.
Keywords: compressible Navier-Stokes equations, ALE method, dynamical elasticity equations, discontinuous Galerkin method, conforming finite elements, weak and strong coupling, vibrations of vocal folds.

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A. Chertock: “A simple eulerian finite-volume method for compressible fluids in domains with moving boundaries”

In this talk, I will present a simple new Eulerian method for treatment of moving boundaries in compressible fluid computations. The approach is based on the extension of the interface tracking method recently introduced in the context of multifluids. The fluid domain is placed in a rectangular domain of a fixed size, which is divided into Cartesian cells. At every discrete time level, there are three types of cells: internal, boundary and external ones. The internal cells are fully occupied by the gas, the external cells are located outside of the fluid domain and play the role of the so-called ghost cells, while the boundary cells form a thin layer between the internal and external ones. The numerical solution is evolved in internal cells only while the boundary cells are used to approximate the point values of the solution at the edges of these cells, required for the numerical flux computations. These point values are obtained using the solid wall extrapolation followed by the interpolation in the phase space (by solving the Riemann problem between the internal cell averages and the extrapolated ones). The proposed computational framework is general and may be used in conjunction with one’s favorite finite-volume method. I will illustrate the robustness of the proposed approach on a number of one- and two-dimensional numerical examples.

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G. Rusnakova: "Kinematical splitting of fluid flow in moving domains"

In this contribution we will consider a coupled fluid-structure interaction problem of non-Newtonian fluid in a moving domain. Fluid flow is decribed by the conservation laws including non-linearities in convective and diffusive term. In order to capture the compliance of moving boundary the so-called arbitrary Lagrangian-Eulerian approach is used. The structure elastic movement is decribed by the generalized string model.
We will present a new loosely-coupled fluid-structure interaction algorithm based on the operator splitting strategy. The algorithm is extended for non-Newtonian shear-dependent fluids and stenotic geometries. Moreover, we allow the use of the second order splitting methods. We will show the crucial role of the geometric conservation law condition in the stability analysis of the coupled problem. The efficiency of the proposed numerical scheme will be compared with the global iterative method. Experimental analysis of hemodynamical wall parameters will show their dependence on non-Newtonian rheology as well as the geometry. This work has been done in cooperation with M. Lukacova and A. Hundertmark.

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D. Praetorius: “Convergence and quasi-optimality of adaptive boundary element methods ”

A posteriori error estimation and adaptive mesh-refinement have themselves proven to be effective tools in scientific computing. In practice, these algorithms lead to quasi-optimal convergence behaviour with respect to the number of degrees of freedom. In the last decade, these empirical observations have been mathematically explained for adaptive finite element methods (AFEM) and elliptic model problems. However, even plain convergence of AFEM has been a major topic in research. For boundary integral equations, the fractional Sobolev spaces and the non-local boundary integral operators involved lead to further technical difficulties. Consequently, for adaptive boundary element methods (ABEM) much less is known and even convergence of ABEM has essentially been open.

In our talk, we consider an adaptive algorithm in the context of the boundary elment method (ABEM), where the mesh-refinement is driven by the weigthed-residual error estimator. We discuss the estimator reduction concept to prove convergence of this ABEM. Moreover, we provide the mathematical frame to prove quasi-optimal convergence rates. Emphasis is laid on the fact that the efficiency of the error estimator (lower bound) is not needed to prove quasi-optimality of the adaptive algorithm, but only to characterize the approximation class involved.

The talk is based on joint work with Markus Melenk, Michael Feischl, and Michael Karkulik (all from TU Wien)

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M. Semplice: "Grid adaptivity for systems of conservation laws"

The approximation of solutions of conservation laws over large domains, especially when complex wave structures emerge, depend crucially on the grid size that one can work with. In this respect, the possibility of concentrating the computing power on the important features of the solution, adapting the grid to the solution itself as the evolution of shocks and waves proceeds, can yield important computational savings.

In this talk I will describe the work I've been doing with G. Puppo on grid adaptive techniques for systems of conservation laws. I will touch upon the obvious questions that arise when using cells of different sizes in a high order finite volume scheme: how to maintain conservativity, how to select the timestep respecting the CFL condition, how to ensure that shocks or other waves will not be deformed or reflected crossing a discontinuity in the grid.

In particular, we present a numerical method for a system of conservation laws, based on a single (nonuniform) grid, stored in a tree. The depth of the nodes in the tree is linked to the cell size and visiting the leaves of the tree means visiting all the currently active cells of the grid. Time advancement is achieved either globally (selecting a timestep satisfying a global CFL condition) or with local timestepping techniques, with a timestep that varies from cell to cell, while maintaining conservativity (Puppo&Semplice, CiCP 2011).

The second step is to introduce grid adaptivity, for which we employ the entropy residual as an indicator. We discuss the accuracy on smooth and nonsmooth problems, using a number of numerical fluxes and flux limiters. Time permitting, recent 2D applications developed with the DUNE library will be shown. Join work with G. Puppo.

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