Lagrangian and Eulerian aspects of turbulent flows with dilute polymers - some representative results

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Outline

- Background
- Motivation
- Experimental study - 3D-PTV
- Lagrangian/Eulerian results
- Discussion
Motivation is both basic and practical

- Drag reduction has been studied since 1948 Toms effect
- Body of literature is huge, important contributions of the present in this room

*The turbulence which occurs in the presence of drag-reducing additives is different from the turbulence which occurs in the solvent alone. Indeed, in some cases of very dilute polymer solutions, the anomalous (i.e. less dissipative) turbulence is probably the only detectable non-Newtonian effect.* McComb 1990
Not only drag reduction

\[
\frac{1}{\sqrt{f}}
\]

\[
\log(Re \sqrt{f})
\]

MDR asymptote

Prandtl - Karman law

laminar

http://lautaro.bionik.tu-berlin.de

Water

30 ppm POLYOX
Phenomenology of polymer effects

- Fluctuating and complex strain field is necessary to “turn the effect on”
- Reaction back changes the field of strain, e.g. resistance to large strain, suppression of strong events, bursts
- The flow could be considered intermittently rheological and not evenly distributed (networks)
- The polymer drag reduction is not necessarily associated with suppression of turbulence, but with qualitative changes of some of its structure and production. In other words, there exist turbulent flows with strongly reduced drag and consequently dissipation and strain.
Motivation

- **Turbulent flows with polymer solutions** - important example where the **Lagrangian approach is unavoidable**:
  1. The **material elements** (Lagrangian objects) are **not passive**;
  2. There are no equations reliably describing flows of polymer solutions (such as NSE for Newtonian fluids).

*There is a need for Lagrangian experimentation with such turbulent flows (and any other active additives), but ....*
• **Lagrangian** methods alone are **limited** - there is a necessity of **Eulerian** approaches in parallel:

1. The fluid particle acceleration $a \equiv Du/Dt$ (Lagrangian) and the Eulerian components.
2. Evolution of small scales via Lagrangian approaches using strain and vorticity in Eulerian form.
3. Dealing with the material elements one needs again quantities such as strain and vorticity in Eulerian form.
4. Eulerian approaches are needed for large scale issues as Reynolds stresses and TKE production.
5. Direct interaction of small and large scales may be exhibited by mixed quantities: $a_L = \omega \times u$
Representative results

The results presented will cover the following topics:

1. Accelerations
2. Velocity derivatives
3. Material elements
4. Large scale stuff (RS and TKE)
5. Direct interaction of SS and LS as may be exhibited by $a_L = \omega \times u$ and perhaps something else available $(\omega \cdot u)$ and (doubtfully) in the spirit of Brasseur.
Experimental setup

Light from 20W CW Ar-Ion laser

Stereoscopic view from four CCD cameras

Observation volume of 10 x 10 x 10 mm

Schematic drawing of a disk with 6 baffles

Front view

Top view

Liberzon et al. (2005).
Experimental method
Experimental principles
The important thing is that we measure directly the full gradient tensor along the particle trajectories: \( \partial u_i / \partial x_j \) and its evolution in time.
Quality checks: Lagrange vs Euler

Lagrangian acceleration, the material derivative of velocity vector, \(a\),

\[
a \equiv \frac{Du}{Dt} = \frac{\partial u}{\partial t} + (u \cdot \nabla)u = -\frac{1}{\rho} \nabla p + \nu \nabla^2 u
\]

is studied in conjunction with its physically important Eulerian decompositions:

\[
a = a_l + a_c = a_\parallel + a_\perp = a_L + a_B
\]

where \(a_l = \frac{\partial u}{\partial t}\) is the local acceleration, \(a_c = (u \cdot \nabla)u\) is the convective acceleration, \(a_\parallel = (a \cdot u)u\) is the acceleration component parallel to the velocity vector, \(a_\perp = a - a_\parallel\) is the acceleration component normal to the velocity vector, \(a_L = \omega \times u\) is the Lamb vector and \(a_B = \nabla (u^2/2)\);
Joint PDF of $a$ and $a_l + a_c$

Solid line - water, dashed line - polymers
PDFs of the magnitudes of the acceleration vector (|\mathbf{a}|) and of its components for water (solid lines) and polymer (dashed lines). (left) dimensional form (right) dimensionless form, normalized with $\varepsilon^{3/2} \nu^{-1/2}$.
PDFs of acceleration components (cont.)

![Graph showing PDFs of acceleration components]
PDFs of acceleration components (cont.)
Ratios of PDFs of polymer to water

Ratios of polymer/water pdfs

Ratios of polymer/water pdf

\[ a_{\perp}, a_{L}, a_{\parallel}, a_{\text{B}} \]
Alignment of $a_l$ and $a_c$
Lagrangian information on the evolution of material elements

Infinitesimal material lines, \( l_i \) evolve according to a purely kinematic equation:

\[
\frac{Dl_i}{Dt} = W_i
\]

\[
W_i = l_j s_{ij} + (1/2) \varepsilon_{ijk} l_k \equiv (\mathbf{s} \cdot \mathbf{l})_i + (1/2)(\mathbf{\omega} \times \mathbf{l})_i
\]

Term 1) Change of magnitude of \( \mathbf{l} \), and Term 2) the tilting of \( \mathbf{l} \). More details in Liberzon et al. PoF (2005)
Stretching related quantities - Cauchy-Green tensor eigenvalues

\[ \ell_i(t) = B_{ij}(t) \ell(0), \quad dB_{ij}/dt = (\partial u_i/\partial x_k) B_{kj}, \quad B_{ij}(0) = \delta_{ij}, \quad W_{ij} = B_{ik}B_{kj} \]
Stretching dynamics of infinitesimal material lines through a single tensor

\[ \ell_i(t) = B_{ij}(t) \ell_j(0), \quad dB_{ij}/dt = (\partial u_i/\partial x_k) B_{kj} \quad B_{ij}(0) = \delta_{ij} \]

\[ \ell_i \ell_j s_{ij} = B_{ik} B_{jm} s_{ij} \ell_k(0) \ell_m(0) \equiv T_{km}(t) \ell_k(0) \ell_m(0) \]

\[ T_{km}(t) \ell(0) \ell_m(0) = \ell^2(0) \left[ \mathcal{T}_i \cos^2(\ell(0), \tau_i) \right] \]

\[ \langle \ell_i \ell_j s_{ij} \rangle = \langle \mathcal{T}_i \rangle \times \langle \cos^2(\ell^0, \tau_i) \rangle = \frac{1}{3} \langle \ell^2(0) \rangle \langle \mathcal{T}_1 + \mathcal{T}_2 + \mathcal{T}_3 \rangle \]

1. trace \( tr(\mathcal{T}) \) is positive on average
2. empirically found that one eigenvalue is three orders of magnitude larger than others
3. it was shown to be strongly reduced in dilute polymers flow
Strong reduction of the “stretching eigenvalue” in polymers

FIG. 5. PDF of the first eigenvalue $Y_1$ of the $T$ matrix for water (solid lines) and polymer solution (dashed lines) for different time moments.
Stretching rates
Notice the “delay” of polymer stretching rate - could explain the resistance to strong strain via mis-alignment or tilting.
Thin lines - water, thick lines - polymers. Solid lines are for $\tau = 0$, dashed for $\tau = 1$ and dash-dot lines are $\tau = 3$. 
Large scale effects, TKE production
PDF of alignment

\[ \cos(-\langle u_i u_j \rangle, S_{ij}) \]

- Water, smooth
- Polymer, smooth
- Water, baffles
- Polymer, baffles
Lagrangian information is crucial in the case of dilute polymers (and probably particles, bubbles, fibers, colloids, etc.)

Eulerian information is crucial, maybe because our Lagrangian formulation is very limited and we need dynamics explained by strain, vorticity, etc.

Mixing Lagrangian and Eulerian information could help to get some new ideas.
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