WPI 2012
Lagrange vs Euler

Luca Biferale
Dept. of Physics, University of Rome “Tor Vergata”
INFN & ICTR
biferale@roma2.infn.it
Lagrangian point sources

Eulerian

Lagrangian Turbulence

Luca Biferale
Università di Tor Vergata, Roma, Italy
Guido Boffetta
Università di Torino, Torino, Italy
Antonio Celani
INL-CNRS, Nice, France
Alessandra Lanotte
ISAC-CNR, Lecce, Italy
Federico Toschi
IAC-CNR, Roma, Italy

Lagrangian-Eulerian
Point-vortices + tracers

If $N > 4$ the system is not integrable

$\lambda_E > 0$

Tracers close to a vortex

Tracers far from vortex

Lyapunov exponents
\[ \delta_r u = [u(x + r) - u(x)] \quad \delta_r v = [u(x(t + \tau)) - u(x(t))] \]

\[
\begin{cases}
\delta_r u_L = [u(x + r) - u(x)] \cdot \hat{r} \\
\delta_r u_T = [u(x + x) - u(x)] \cdot \hat{n}
\end{cases}
\]
OCCAM’S RAZOR

"Entia non sunt multiplicanda praeter necessitatem”
(Entities should not be multiplied unnecessarily)

\[ \tau \sim r^{2/3} \]

\( \gamma(k) = k^{2/3} \)

- Locality of interactions (problem for 2D turbo)
- only one time scale (problem for MHD)
Intermittency - Eulerian

Figure 4
Snapshot of the intensity distributions of (a) the energy-dissipation rate $\varepsilon = \varepsilon/(2v)$ and (b) the enstrophy $\Omega = \omega^2/2$ on a cross section in DNS-ES at $R_\lambda = 675$ in arbitrary units.

Study of High-Reynolds Number Isotropic Turbulence by Direct Numerical Simulation

Takashi Ishihara,1 Toshiyuki Gotoh,2 and Yukio Kaneda1

1Department of Computational Science and Engineering, Graduate School of Engineering, Nagoya University, Chikusa-ku, Nagoya 464-8603, Japan; email: ishihara@comp.nagoya-u.ac.jp
2Department of Scientific and Engineering Simulation, Graduate School of Engineering, Nagoya Institute of Technology, Gokiso, Showa-ku, Nagoya 466-8555, Japan
Intermittency - Lagrangian

Particle trapping in three-dimensional fully developed turbulence

L. Bilotti
Dipartimento di Fisica and INFN, Università degli Studi di Roma “Tor Vergata,”
Via della Ricerca Scientifica 1, 00133 Roma, Italy

G. Boffetta
Dipartimento di Fisica Generale and INFN, Università degli Studi di Torino, Via Pietro Giuria 1, 10125 Torino, Italy

A. Celani
CNR-ING, 1561 Route des Lucioles, 06560 Valbonne, France

A. Lanotte
CNR-ISAC, Str. Prov. Lecce-Monteroni km. 1200, 73100 Lecce, Italy

F. Toschi
Istituto per le Applicazioni del Calcolo, CNR, Viale del Politecnico 137, 00161 Roma, Italy

Lagrangian Properties of Particles in Turbulence

Federico Toschi¹ and Eberhard Bodenschatz²

1Istituto per le Applicazioni del Calcolo, CNR, 00163 Rome, Italy; INFN, Sezione di Ferrara, I-44100 Ferrara, Italy; Department of Physics and Department of Mathematics and Computer Science, Eindhoven University of Technology, 5600 MB Eindhoven, The Netherlands, and International Collaboration for Turbulence Research, email: toschi@iac.cnr.it

2Max Planck Institute for Dynamics and Self-Organization, D-37077 Göttingen, Germany; Laboratory of Atomic and Solid State Physics and Institute of Fluid Dynamics, Cornell University, Ithaca, New York 14853, Institute for Nonlinear Dynamics, University of Göttingen, D-37077 Göttingen, Germany, and International Collaboration for Turbulence Research
IF YOU WANT TO PREDICT EXTREME EVENTS YOU CANNOT NEGLECT INTERMITTENCY
EFFECTS OF INTERMITTENCY (II)

\[ \mathcal{P}(\delta \tau \tilde{u}) \]

\[ \delta \tau \tilde{u} = \frac{\delta \tau v}{\langle (\delta \tau v)^2 \rangle^{1/2}} \]

2048^3

16 Mega particles

\[ F(\tau) = \frac{\langle (\delta \tau v)^4 \rangle}{\langle (\delta \tau v)^2 \rangle^2} \]
\[ \zeta_L(p, r) \stackrel{\text{DEF}}{=} \frac{d \log \langle (\delta_r u_L)^p \rangle}{d \log r} \]

\( p = 2, 4, 6, 8 \)

**In Log-Log All Cows Are Black!**
EULERIAN STATISTICS: LONGITUDINAL VS TRANSVERSE

\[ S^{(p)}_L(r) = \langle (\delta_r u_L)^p \rangle \]

\[ \zeta_L(p, r) = \frac{d \log \langle (\delta_r u_L)^p \rangle}{d \log r} \]

\[ S^{(p)}_T(r) = \langle (\delta_r u_T)^p \rangle \]

\[ \zeta_T(p, r) = \frac{d \log \langle (\delta_r u_T)^p \rangle}{d \log r} \]

LOCAL SLOPES: LONGITUDINAL AND TRANSVERSE:

\[ p = 10 \]

\[ r/\eta \]

\[ p = 8 \]

\[ r/\eta \]

\[ p = 6 \]

\[ r/\eta \]

\[ p = 4 \]

\[ r/\eta \]

\[ p = 2 \]

\[ r/\eta \]
LOCAL SLOPES: LONGITUDINAL AND TRANSVERSE:

\[ \zeta_L(p, r) \rightarrow \zeta_L(p) \quad \zeta_T(p, r) \rightarrow \zeta_T(p) \]

INERTIAL RANGE: LONGITUDINAL VS TRANSVERSE

R. Benzi, L.B. R. Fischer, L. Kadanoff, D. Lamb, F. Toschi
PRL 100, 234503 (2008).

LONGITUDINAL AND TRANSVERSE SCALE DIFFERENTLY

\[ \zeta_L(p, r) = \frac{d \log \langle (\delta_r u_L)^p \rangle}{d \log r} \]

\[ \zeta_T(p, r) = \frac{d \log \langle (\delta_r u_T)^p \rangle}{d \log r} \]

<table>
<thead>
<tr>
<th>p</th>
<th>(\zeta_L^{(p)})</th>
<th>(\zeta_T^{(p)})</th>
<th>(\zeta_L^{(p)}) ref. []</th>
<th>(\zeta_T^{(p)}) ref. []</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.71 ± 0.02</td>
<td>0.71 ± 0.02</td>
<td>0.70 ± 0.01</td>
<td>0.71 ± 0.01</td>
</tr>
<tr>
<td>4</td>
<td>1.29 ± 0.03</td>
<td>1.27 ± 0.05</td>
<td>1.29 ± 0.03</td>
<td>1.26 ± 0.02</td>
</tr>
<tr>
<td>6</td>
<td>1.78 ± 0.04</td>
<td>1.68 ± 0.06</td>
<td>1.77 ± 0.04</td>
<td>1.67 ± 0.04</td>
</tr>
<tr>
<td>8</td>
<td>2.18 ± 0.05</td>
<td>1.92 ± 0.10</td>
<td>2.17 ± 0.07</td>
<td>1.93 ± 0.09</td>
</tr>
<tr>
<td>10</td>
<td>2.50 ± 0.06</td>
<td>2.10 ± 0.20</td>
<td>2.53 ± 0.09</td>
<td>2.08 ± 0.18</td>
</tr>
</tbody>
</table>

Gotoh et al. (PoF 2002)
Figure 4

Snapshot of the intensity distributions of (a) the energy-dissipation rate $\varepsilon = e/(2\nu)$ and (b) the enstrophy $\Omega = \omega^2/2$ on a cross section in DNS-ES at $Re = 675$ in arbitrary units.

CASCADE PROCESS -> LARGE DEVIATIONS -> MULTIFRACTAL MEASURE OR MULTIAFFINE SIGNALS
USE TWO DIFFERENT MULTIFRACTAL $D(h)$ TO FIT SEPARATELY LONGITUDINAL AND TRANSVERSE EULERIAN SCALING

\[ \delta_r u_L \sim r^h; \quad \mathcal{P}_h(r) \sim r^{3-D_L(h)} \]
\[ S_L^{(p)} \sim r^{\zeta_L(p)} \]

\[ \delta_r u_T \sim r^h; \quad \mathcal{P}_h(r) \sim r^{3-D_T(h)} \]
\[ S_T^{(p)} \sim r^{\zeta_T(p)} \]

\[ \zeta_{L,T}(p) = \min_h [ph + 3 - D_{L,T}(h)] \]

Parisi & Frisch (1983)
\langle (\delta_{\tau} u)^3 \rangle \propto \varepsilon r \quad \longleftrightarrow \quad \langle (\delta_{\tau} v)^2 \rangle \propto \varepsilon \tau

Borgas (1993)

EULERIAN MULTIFRACTAL
\delta_{\tau} u \sim r^h
P_{h}(r) \sim r^{3-D_E(h)}

BRIDGE RELATION
\delta_{\tau} u \sim \delta_{\tau} v
\tau \sim r / \delta_{\tau} u

LAGRANGIAN MULTIFRACTAL
\delta_{\tau} v \sim \tau^{1-h}
P_{h}(\tau) \sim \tau^{3-D_E(h) \over 1-h}
\[
\begin{align*}
S_{L}^{(p)}(r) &= \langle (\delta_r u_L)^p \rangle \\
S_{T}^{(p)}(r) &= \langle (\delta_r u_T)^p \rangle \\
\zeta_{L,T}(p) &= \min_h [ph + 3 - D_{L,T}(h)] \\
S_{Lag}^{(p)}(\tau) &= \langle (\delta_\tau u)^p \rangle \sim \tau \zeta_{Lag}(p) \\
\zeta_{Lag}(p) &= \min_h \left[ \frac{ph + 3 - D_{L,T}(h)}{1 - h} \right]
\end{align*}
\]
Ott and Mann experiment at Risø
conventional 3D PTV –
Re~100-300

Pinton et al ENSL
Acoustic/Laser Doppler
tracking -
Re ~800 (single particle
tracking)

Bodenschatz et al at Cornell-
MPI
silicon strip detectors (now
also CCD) Re ~1000-1500

Warhaft et al
experiment at Cornell
Fast moving camera
Re~ 300

Luthi, Tsinober et al
3D PTV and 3D scanning PTV for
velocity gradients

and many others….
F. Toschi & E. Bodenschatz ARFM 41, 375 (2009)

non intrusive tracking down to
\[ \tau \sim \tau_{\eta} \]
\[ \langle (\delta \tau \nu)^p \rangle \sim \epsilon^{p/2} \tau^{p/2} \]

Batchelor parametrization

\[ S_2(\tau) = C_0 \frac{\tau^2}{(c_1 \tau_n^2 + \tau^2)^{(2-z_2)/2}} (1 + c_3 \tau/T_L)^{-z_3}, \]

L.B. and A. Lanotte NSF-ITP-11-103 (2001)

B. Sawford and P.K. Yeung PoF 23, 091704 (2011) & NEXT TALK by Eberhard
\[ \chi_{\tau}(p) = \frac{d \log \langle (\delta_{\tau}v)^p \rangle}{d \log \langle (\delta_{\tau}v)^2 \rangle} \quad \chi_{\tau}(p) \rightarrow \chi(p) \]
INFINITELY-MANY ANOMALOUS SCALING EXPONENTS
(MULTIFRACTAL FIELD, Parisi & Frisch, 1983)

\[ S_p(\tau) = \langle (v(t + \tau) - v(t))^p \rangle \sim \tau^{\zeta_L(p)} \]

in the scaling range:

\[ \zeta_L(\tau, p) \rightarrow \zeta_L(p) \]
WE LEARN ABOUT:
(i) INTERMITTENCY; (ii) UNIVERSALITY; (iii) ANISOTROPY

\[ F(\tau) = \langle (\delta_\tau \upsilon)^2 \rangle (\zeta_L(\tau, 4) - 2) \]


[Phys. Rev. Lett 100, 254504 2008]
WHAT HAPPENS AROUND DISSIPATIVE TIME?

FIG. 1: Log-Lin plot of the local exponent for the fourth moment, \( \zeta(4, \tau) \), averaged over the three velocity components, as a function of the normalised time lag \( \tau/\tau_\eta \). Data sets come from three experiments (EXP) [see table 1] and five direct numerical simulations (DNS) (see table 2). Error bars are estimated out of the spread between the three components, but for EXP1 and EXP3 where only two components have been considered because of large systematic anisotropic effects in the third one. Each data set is plotted only in the time range where the known experimental/numerical limitations are certainly not affecting the results. In particular, for each data set, the largest time lag always satisfies \( \tau < T_\eta \). The minimal time lag is set by the highest fully resolved frequency. The shaded area displays the prediction obtained by the MF model by using \( D_{\tau}(h) \) or \( D_{\gamma}(h) \), with
\[
\begin{align*}
\delta_\tau u &= v_0 \frac{\tau/T_L}{\left[\left(\frac{\tau}{T_L}\right)^\beta + \left(\frac{\tau_\eta}{T_L}\right)^\beta\right]^{\frac{1-2h}{\beta(1-h)}}} \\
\mathcal{P}_h(\tau, \tau_\eta) &\sim \left[\left(\frac{\tau}{T_L}\right)^\beta + \left(\frac{\tau}{T_L}\right)^\beta\right]^{\frac{3-D(h)}{\beta(1-\nu_\eta)}} \\
\delta_\tau v &\sim (\frac{\tau}{T_L})^{\frac{\eta}{1-h}} \\
\delta_\tau v &\sim \tau \frac{\delta_{\tau_\eta} v}{\tau_\eta} \sim a_\tau \\
\end{align*}
\]

but: dissipative time fluctuates (as the dissipative scale): \( \tau_\eta = \frac{\eta}{\delta_\eta v} \)

\[
\tau_\eta(h) \sim Re^\frac{2(h-1)}{1+h}
\]
\[ \zeta_p(\tau) = \frac{d \log (S_p(\tau))}{d \log (S_2(\tau))} \]
$\mathcal{P}(\tilde{a}) \sim \int_{h \in I} dh a^{\frac{h-D(h)}{3}} \nu^{\frac{7-2h-2D(h)}{3}} L_0^{D(h)+h-3} \sigma^{-1} \exp \left( \frac{a^{2(1+h)}}{3} \nu^{\frac{2(1-h)}{3}} L_0^{2h} \frac{1}{2\sigma_v^2} \right)$
\( \langle a^2 | v^2 \rangle \)

\[ u^{4.57} \]

Multifractal prediction


FIG. 4 (color online). Normalized conditional acceleration variance \( \langle a^2 | u \rangle / \sigma_u^2 \) for \( R_A = 690, 485, 285 \), circles, triangles, and squares, respectively. Solid lines are the fit (3).


OPEN QUESTIONS (FAILURES): Effects of Inertia

<table>
<thead>
<tr>
<th></th>
<th>bubble</th>
<th>tracer</th>
<th>heavy</th>
</tr>
</thead>
</table>

- **bubble**: [Image of bubble simulation]
- **tracer**: [Image of tracer simulation]
- **heavy**: [Image of heavy simulation]
Eqns of motion for a single particle

- Small particles
- Small Reynolds numbers (on the particle radius)
- Undeformable
- Small volume fraction
- Collisionless

\[ m_p \frac{dV_i}{dt} = (m_p - m_f)g_i + m_f \frac{Du_i}{Dt} \bigg|_{X(t)} \]

\[ -6\pi a\mu \left[ V_i(t) - u_i(X(t), t) - \frac{1}{6} a^2 \nabla^2 u_i \bigg|_{X(t)} \right] \]

\[ -\frac{m_f}{2} \frac{d}{dt} \left[ V_i(t) - u_i(X(t), t) - \frac{1}{10} a^2 \nabla^2 u_i \bigg|_{X(t)} \right] \]

\[ -6\pi a\mu \int_0^t ds \left( \frac{d/ds \left[ V_i(s) - u_i(X(s), s) - \frac{1}{6} a^2 \nabla^2 u_i \bigg|_{X(s)} \right]}{\sqrt{\pi \nu (t-s)}} \right) \]

\[ \frac{a(u - V)}{\nu} \ll 1 \quad a \ll \eta \]

Buoyancy + fluid acceleration

Stokes drag

Added mass

Basset-history terms
Simplified limit

\[ \frac{dX}{dt} = V \]

\[ \frac{dV}{dt} = \beta \frac{Du(X,t)}{Dt} + \frac{u(X,t)-V}{\tau_p} + (1 - \beta)g \]

**Stokes number**

\[ St = \frac{\tau_p}{\tau_f} \]

**Density contrast**

\[ \beta = \frac{3\rho_f}{\rho_f + 2\rho_p} \]

**Reynolds**

\[ Re = \frac{UL}{\nu} \]
Validity of assumption $\frac{a}{\eta} < 1$

$$St = \frac{\tau_p}{\tau_f}$$

$$\frac{dX}{dt} = V$$

$$\frac{dV}{dt} = \beta \frac{Du(X,t)}{Dt} + \frac{u(X,t) - V}{St}$$

$$\beta = \frac{3\rho_f}{\rho_f + 2\rho_p}$$
Preferential Concentration

Okubo–Weiss parameter $Q$ is the determinant of the strain matrix

$p_2(r) \sim r^{D_2 - 1}$

$$\Delta = \left( \frac{\det[\hat{\sigma}]}{2} \right)^2 - \left( \frac{\text{Tr}[\hat{\sigma}^2]}{6} \right)^3 \Delta \leq 0$$

$$\Delta > 0$$

Okubo–Weiss parameter $Q$ is the determinant of the strain matrix

$$\sigma_{ij} = \frac{\partial u_i}{\partial x_j}$$
Acceleration: pdf(a) vs. St

St=0, 0.16, 0.37, 0.58, 1.01, 2.03, 3.33 at Re_{\infty}=185

Q: how to include inertia in Multifractal phenomenology? Nobody knows

Local slopes ($\text{order } 6$ vs. $\text{order } 2$)

Increase inertia

Figure from: On the effects of vortex trapping on the velocity statistics of tracers and heavy particle in turbulent flows

\[
\frac{d \log(S_6(\tau))}{d \log(\tau)}
\]
Role of vortex filaments vs fluctuations of dissipative scales

Light/heavy particles are trapped/ejected inside/outside vortex filaments: they fill more/less the fluctuations of the dissipative scale.

\[ \frac{d \log(S_4(\tau))}{d \log(\tau)} \]

Mutifractal AND fluctuations of dissipative scale

Universality of inertial range with respect to ‘huge’ small-scale effects
- Kraichnan et al: superposition of random vortex filaments: $k41$ scaling with longitudinal=transverse scaling.
- Belin, Maurer, Tabeling & Willaime: filaments transition (statistical instability) at Re $\sim 700$
- Chorin: collection of sel-avoiding vortex filaments -> fractal structure
- Passot Politano et al: influence of vortex filaments on the energy spectrum
- Migdal: loop turbulence, statistics driven by velocity circulation

8.9.2 Statistical signature of vortex filaments: dog or tail?

Having identified ‘simple’ geometric objects, the vortex filaments, in turbulent flows, it is natural to ask if any of the known statistical properties of turbulence can be thus explained. Are the vortex filaments the *dog* or the *tail*? In the former case, they would be essential to explain the energetics and the scaling properties of high-Reynolds-number flow. In the latter case, they would have only marginal signatures, for example on the tails of p.d.f.s of various small-scale quantities and on the exponents $\zeta_p$ for large $p$. 

Frisch: Turbulence, Cambridge Univ. Press, 1995
IS FORWARD ENERGY CASCADE THE END OF THE STORY IN 3D?

CAN WE DISENTANGLE DIFFERENT PHYSICAL MECHANISM LEADING TO FORWARD/BACKWARD ENERGY TRANSFER IN 3D NS?
The nature of triad interactions in homogeneous turbulence

Fabian Waleffe
Center for Turbulence Research, Stanford University–NASA Ames, Building 500, Stanford, California 94305-3030

(Received 24 July 1991; accepted 22 October 1991)

\[ u(k) = u^+(k)h^+(k) + u^-(k)h^-(k) \]

\[ h^\pm = \mathbf{\hat{v}} \times \mathbf{\hat{k}} \pm i\mathbf{\hat{v}} \]

\[ \mathbf{\hat{v}} = z \times k / ||z \times k|| \]

\[ ik \times h^\pm = \pm k h^\pm \]

\[
\begin{align*}
E &= \sum_k |u^+(k)|^2 + |u^-(k)|^2; \\
H &= \sum_k k(|u^+(k)|^2 - |u^-(k)|^2).
\end{align*}
\]
\[ u^{s_k}(k,t) \quad (s_k = \pm 1) \]
\[
\frac{d}{dt} u^{s_k}(k) + \nu k^2 u^{s_k}(k) = \sum_{k+p+q=0} \sum_{s_p,s_q} g_{k,p,q} (s_p p - s_q q) \times [u^{s_p}(p)u^{s_q}(q)]^*. \tag{15}
\]

Eight different types of interaction between three modes \( u^{s_k}(k) \), \( u^{s_p}(p) \), and \( u^{s_q}(q) \) with \(|k| < |p| < |q|\) are allowed according to the value of the triplet \((s_k,s_p,s_q)\).

\[
\dot{u}^{s_k} = r (s_p p - s_q q) \frac{s_k k + s_p p + s_q q}{p} (u^{s_p} u^{s_q})^*,
\]
\[
\dot{u}^{s_p} = r (s_q q - s_k k) \frac{s_k k + s_p p + s_q q}{p} (u^{s_q} u^{s_k})^*,
\]
\[
\dot{u}^{s_q} = r (s_k k - s_p p) \frac{s_k k + s_p p + s_q q}{p} (u^{s_k} u^{s_p})^*.
\]
TRIADIC INTERACTION IN WHOLE NAVIER-STOKES EQUATIONS
\[ \begin{aligned}
E &= \sum_k |u^+(k)|^2 + |u^-(k)|^2; \\
H &= \sum_k k(|u^+(k)|^2 - |u^-(k)|^2).
\end{aligned} \]
TRIADIC INTERACTION IN WHOLE NAVIER-STOKES EQS

MAINLY FORWARD

\[ u^-(k) \to u^+(p) \]
\[ u^+(q) \to u^-(p) \]

\[ u^-(k) \to u^-(q) \to u^+(p) \]
TRIADIC INTERACTION IN WHOLE NAVIER_STOKES EQS

\[ u^-(k) \rightarrow u^+(p) \rightarrow u^+(q) \]

\[ u^-(k) \rightarrow u^+(p) \rightarrow u^-(q) \]
\[
\mathcal{P}^\pm \equiv \frac{\mathcal{H}^\pm \otimes \mathcal{H}^\pm}{\mathcal{H}^\pm \cdot \mathcal{H}^\pm}.
\]
\[
\nu^\pm(x) \equiv \sum_k \mathcal{P}^\pm u(k);
\]
\[
u^+(k)h^+(k) + u^-(k)h^-(k)
\]

**LOCAL BELTRAMIZATION (IN FOURIER)**

\[
\partial_t v^+ + \mathcal{P}^+ B[v^+, v^+] = \nu \Delta v^+ + f^+
\]

\[
\frac{d}{dt} u^s_k(k) + \nu k^2 u^s_k(k) = \sum_{k+p+q=0} \sum_{s_p, s_q} g_{k,p,q}(s_p p - s_q q)
\]

\[
\times [u^{s_p}(p)u^{s_q}(q)]^*.
\]

\[
s_p = s_q = s_k = +
\]
\[ \begin{align*}
E &= \sum_k |u^+(k)|^2 + |u^-(k)|^2; \\
H &= \sum_k k(|u^+(k)|^2 - |u^-(k)|^2).
\end{align*} \]
MULTI-TIME MULTI-SCALE CORRELATION FUNCTIONS

\[ C(R, r \mid \tau) = \langle |\delta_r u(x(t + \tau), t + \tau)||\delta_R u(x(t), t)|^2 \rangle \]

\[ C_{p, q}(R, r \mid \tau) = \langle |\delta_r u(x(t + \tau), t + \tau)|^p |\delta_R u(x(t), t)|^q \rangle \]

\[ T(R) = \int_0^\infty d\tau C(R, R \mid \tau)/C(R, R \mid 0) \]

\[ T_{p, q}(R) = \int_0^\infty d\tau C_{p, q}(R, R \mid \tau)/C_{p, q}(R, R \mid 0) \sim R^{2p/3 + \delta_{p, q}} \]
1. EULERIAN-LAGRANGIAN BASED ON THE SIMPLEST (OCCAM’S RASOR) PRINCIPLE IS GOOD ALSO FOR INTENSE FLUCTUATIONS (NOT KNOWN FOR VERY INTENSE ONES)

2. IS IT THE END OF THE STORY: NO (2D, MHD, SHEAR, ETC…)

3. WHY LAGRANGIAN SCALING IS SO POOR (ONLY ESS UP TO NOW)

4. WHAT HAPPENS WHEN TOPOLOGY PLAYS A KEY ROLE: INERTIAL PARTICLES

5. WHAT ABOUT MULTI-TIME MULTI-SCALE
Thanks to (order of appearance)

M. Cencini
F. Toschi
G. Boffetta
A. Celani
A.S. Lanotte
B. Devenish
J. Bec
A. Scaglierini
E. Calzavarini
R. Benzi
L.P. Kadanoff
B. Fisher
D. Lamb
E. Bodenshatz
N. Ouellette
H. Xu
G. Falkovich
A. Pumir